Investing for the Rest of Us:

Low Cost, Globally Diversified Dynamic Asset Allocation Using Simple Measures of Value and Momentum to Achieve Superior Long Term Returns

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1) Introduction

The consensus advice to investors for whom investing is not a full time profession is this: buy a diversified portfolio of risky assets and hold for the long term, while setting aside enough, parked safely in low risk assets, to be able to weather the periodic market downturns. Explicit in this counsel is the view that the non-professional investor should not expect to beat the market, either by picking individual investments that will outperform, or even by identifying investment managers who can pick such investments and generate market beating returns. In addition to accepting that we cannot beat the market, this passive approach to investing also requires us to believe that the expected returns offered by the various assets are fair compensation for the risk[2] we bear in holding them.

Most investors have a visceral aversion to a fully passive strategy- we feel we ought to be able to do better. Most investors are reluctant to accept the efficient markets hypothesis as an article of faith. We feel that we should be able to figure out whether it is a particularly good time or bad time to be invested in various risky assets. Sooner or later, the vast majority of us give up on a fully passive strategy because we are overcome with a feeling that certain assets are either due for a correction or, alternatively, that they cannot fail to appreciate. Unfortunately, there is a significant body of empirical research indicating that, on average, non-professional investors (and professional ones too) are not very good at timing their investments to take advantage of the ebb and flow of expected returns. There are a number of studies that conclude that individual investors who actively manage their equity portfolios wind up underperforming a passive, indexed portfolio by multiple percentage points per year. [3]

On the other hand, critics of passive investing have coined the phrase "the tyranny of indexing" to express the shortcomings of the approach. They ask how it can make sense to hold a portfolio with weights determined by market values in the face of recurring bubbles and panics. They hasten to point out that in 1989, when the Japanese stock market was trading at close to 100 times

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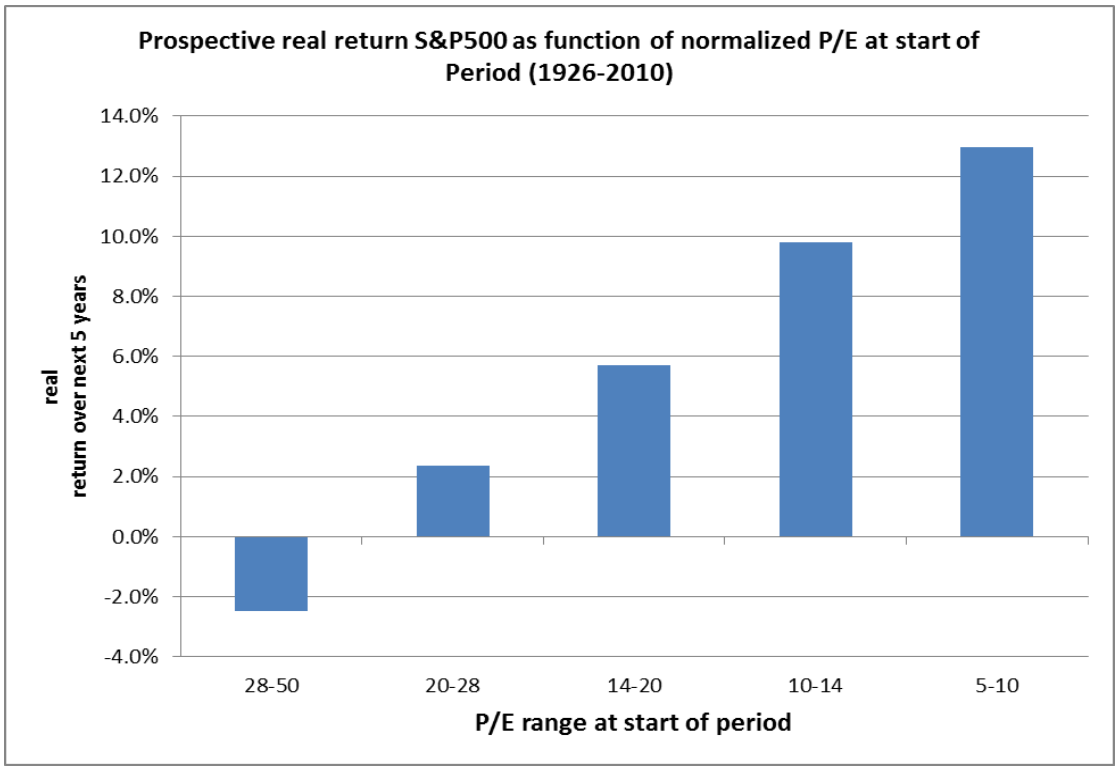
[2] Including the degree of correlation with other risky assets.
[3] See work by Brad M. Barber and Terrance Odean, and also www.dalbar.com , among many others.

earnings[4], a passive index portfolio of global equities would have had roughly 40% allocated to Japanese equities. The result of being invested in perhaps the most overvalued stock market of all time was not pretty; a hundred dollars invested in the Japanese stock market in early 1989 would be worth about 37 dollars today, more than 20 years later.[5] Unfortunately, the Japanese equity bubble was not an isolated event. At the start of the new millennium, the US stock market was valued at roughly 50 times its earnings, and 10 years later we find that 100 dollars invested at that valuation would be worth about 65 dollars today.

Is it really as easy as all that? Cut back on your holding of an asset when it is expensively valued and own more when it is cheap? The answer seems to be yes, with some qualifications. The chart below has appeared, in various formats, in the popular press and in numerous books and articles on investing[6]. It shows quite clearly that when the price/earnings ratio is high (read: stocks are expensive) the next five years of returns are disappointing, and when the P/E ratio is low, subsequent returns tend to be terrific.



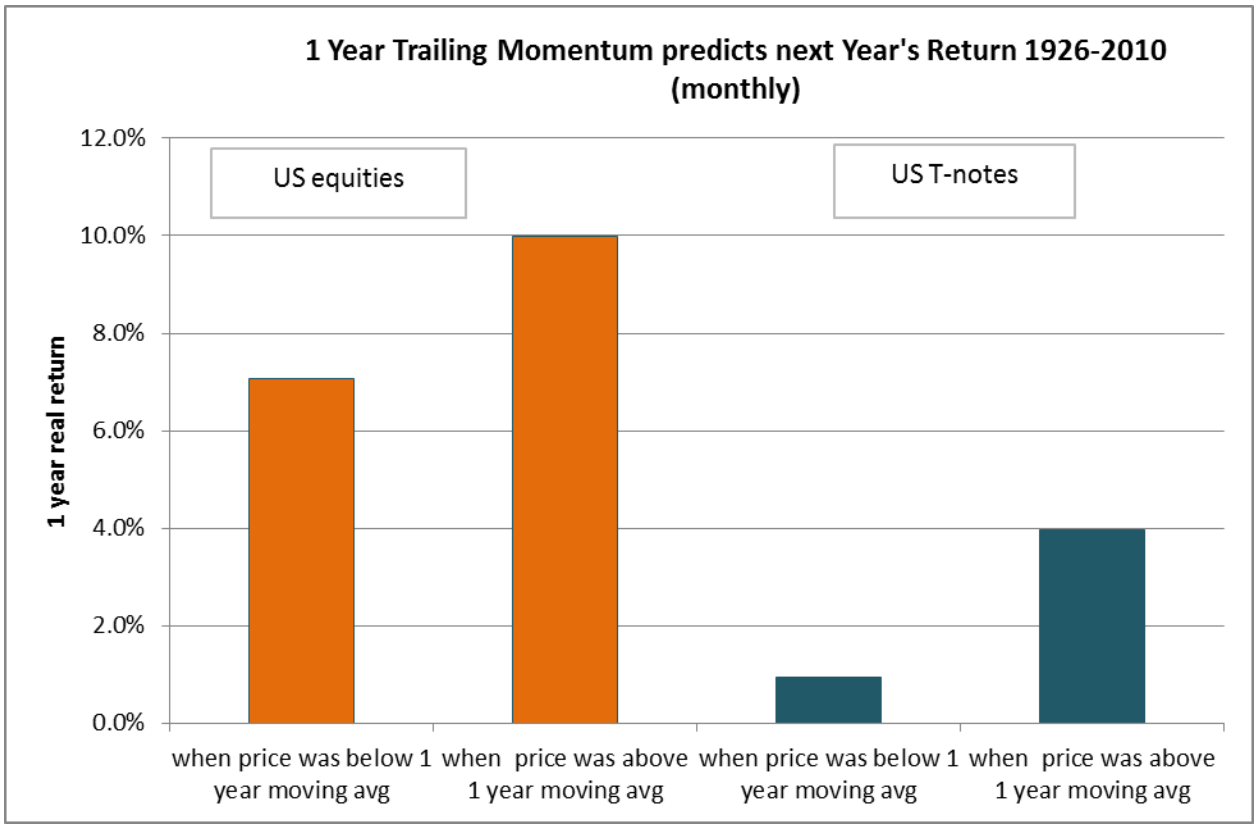
In addition to increasing and decreasing our investments based on valuations, which is surely the most sensible way we can think of to regulate our investing, we believe that giving some respect to market trends helps to increase long term returns, while also decreasing risk.

[4] Earnings means the average of the past 10 years of earnings of all companies in that index. This measure of earnings is often called the Ben Graham earnings (when applied to nominal historical earnings) or Shiller earnings (when applied to inflation adjusted historical earnings), after the economists who popularized this measure. We are using inflation adjusted earnings, and hence we are using the Shiller metric.

[5] In inflation adjusted terms, including reinvested dividends; and from here on, all returns are reported inflation adjusted and including dividends unless otherwise stated.

[6] For example, *Irrational Exuberance*, Robert Shiller (2001).

As can be seen from the chart below, when prices have risen over the past year, it has been more likely for prices to go even higher over the next year, and vice versa. The two bars on the left of the chart show the next year's return for US equities when the price[7] at the beginning of the period is below or above the past year's inflation adjusted average price, and the two bars on the right show the same statistics for US 10 year Treasury bonds. Of course, this experience is not consistent with the idea that markets follow a random walk, a finding which may be troubling to some investors. In practice, mixing a bit of trend following together with a discerning eye towards value very often results in simply a more deliberate and cautious movement of the portfolio to assets as they cheapen, and away from them as they get more expensive. Trend following turns out to be complementary to a value oriented investment style in a number of ways that we will discuss more fully later in this paper.



Surprisingly, we have not found any research that looks at what would have resulted from translating these relationships into easily implementable, non-leveraged, long-only investment strategies using widely available, low cost index funds for the ordinary, non-professional investor.[8]

[7] More precisely, by price we really mean the total return index for that asset.

[8] A few papers that are closely related to our research are: Value and Momentum Everywhere, Asness, Moskowitz, Pedersen (2009), Global Tactical Cross Asset Allocation: Applying Value and Momentum Across Asset Classes, Blitz and Van Vliet,

This research note aims to do just that, and a bit more. It will explore these relationships more broadly, looking at global equity markets, government and corporate bond markets, real estate and commodity markets, over various horizons. In contrast to much of the existing literature on value and momentum, we will focus on their application to asset classes, rather than to individual securities (stocks and bonds).

In agreement with other, more rigorous research by academics and practitioners, we find that prospective returns do have some predictability based on very simple and intuitive metrics, designed to measure underlying value and market trends. Adjusting the allocation of assets in line with changes in each asset's expected return can lead to long run returns that are superior to passive buy and hold strategies, and, importantly, have lower risk.[9]

We will explain this simple, intuitively appealing, low cost investment strategy in sufficient detail for the interested reader either to implement the strategy as is, or to improve and customize the approach for their own particular circumstances. We hope that this value-informed, common sense strategy will be straightforward for investors to embrace, and, of paramount importance, to stick to.

The historical studies we will describe in detail in the body of this paper cover two reasonably long periods[10]:

1) End of January 1926 to end of February 2010 (~84 years), and
2) End of December 1974 to end of February 2010 (~35 years).

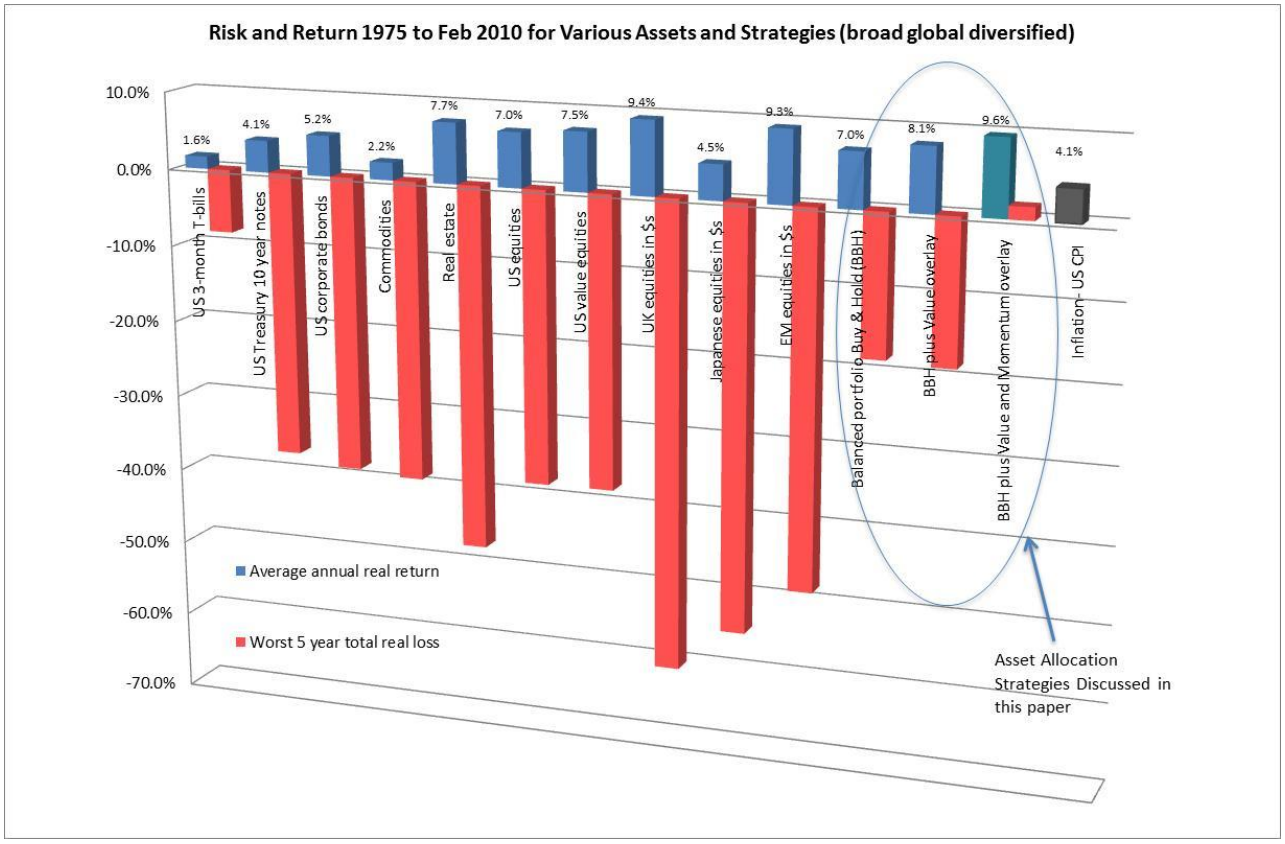
The following two charts provide a summary of what we found[11]. The final three pairs of columns on the right hand side show the returns for strategies that we are describing in this paper.

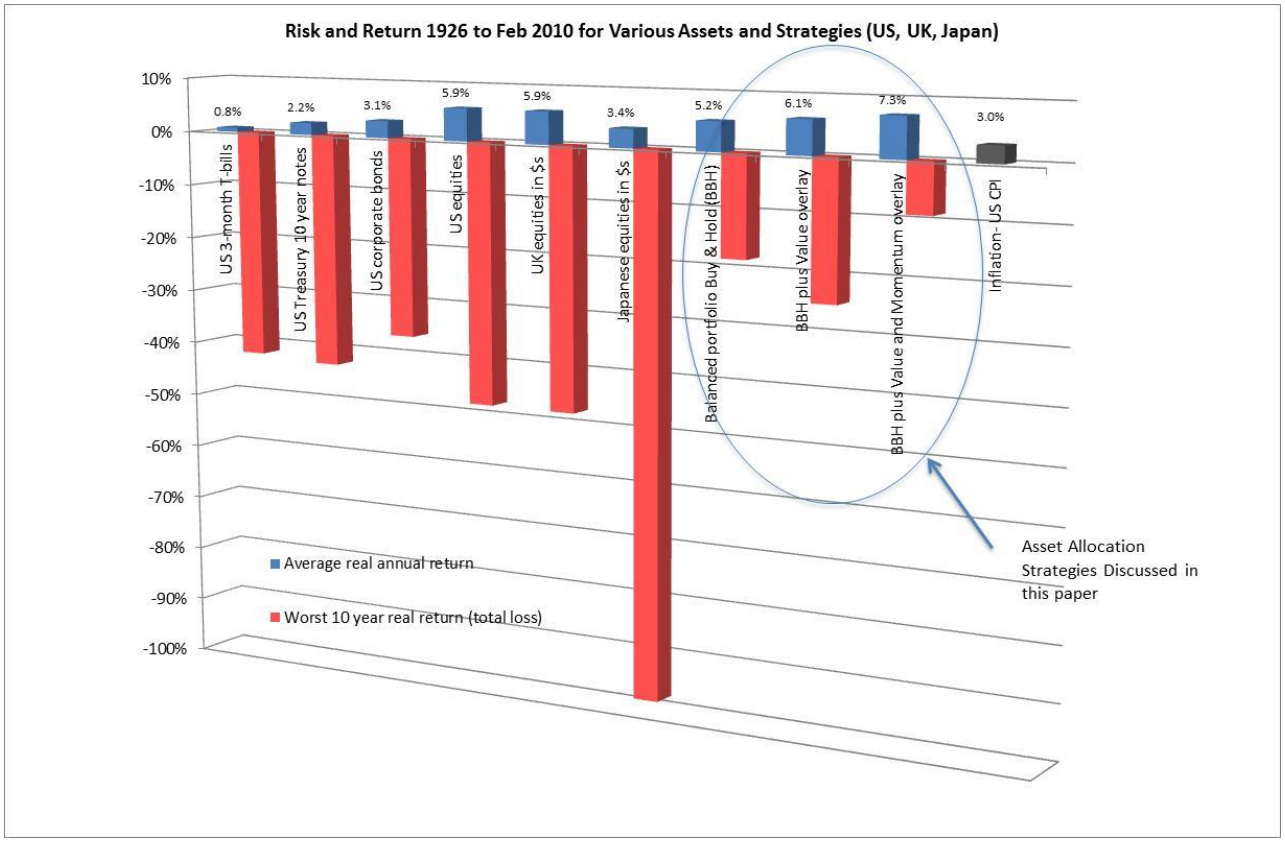
(2008), and Global Tactical Asset Allocation, GSAM (2003). Also, the book, Expected Returns: An Investor's Guide to Harvesting Market Rewards by Antti Ilmanen (2011) is a source of much inspiration for the ideas in this paper.

[9] Although there are many different definitions of risk, which we will explore in greater detail later in this paper, at this point, we are thinking of risk as most commonly conceived- worst case loss over some horizon or, for the more statistically inclined, annual standard deviation.

[10] In fact, we used data starting five years before the start of these two periods in order to have a basis for valuation and momentum that requires a period of data before the start of the historical simulation.

[11] We must admit that we were not terribly surprised by what we found, and perhaps "found" or "discovered" are not appropriate terms to use when it comes to such historical studies. We will talk more about "data mining" later, but the fact that we were not surprised is not surprising given that we couldn't help but have some prior familiarity with the historical record used as our sample data.


Risk and Return 1975 to Feb 2010 for Various Assets and Strategies (broad global diversified)
10.0%
0.0%
-10.0%
-20.0%
-30.0%
-40.0%
-50.0%
-60.0%
-70.0%
1.6%
4.1%
5.2%
2.2%
7.7%
7.0%
7.5%
9.4%
4.5%
9.3%
7.0%
8.1%
9.6%
4.1%
US 3-month T-bills
US Treasury 10 year notes
US corporate bonds
Commodities
Real estate
US equities
US value equities
UK equities in $s
Japanese equities in $s
EM equities in $s
Balanced portfolio Buy & Hold (BBH)
BBH plus Value overlay
BBH plus Value and Momentum overlay
Inflation- US CPI
Average annual real return
Worst 5 year total real loss
Asset Allocation Strategies Discussed in this paper


Risk and Return 1926 to Feb 2010 for Various Assets and Strategies (US, UK, Japan)
10%
0%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
-80%
-90%
-100%
0.8%
2.2%
3.1%
5.9%
5.9%
3.4%
5.2%
6.1%
7.3%
3.0%
US 3-month T-bills
US Treasury 10 year notes
US corporate bonds
US equities
UK equities in $s
Japanese equities in $s
Balanced portfolio Buy & Hold (BBH)
BBH plus Value overlay
BBH plus Value and Momentum overlay
Inflation- US CPI
Average real annual return
Worst 10 year real return (total loss)
Asset Allocation Strategies Discussed in this paper

First, it was comforting, but no surprise, that our research results were consistent with three of the most widely held tenets of investment theory and practice:

1) *Portfolios with a healthy weight in equities have delivered robust returns to investors*. A portfolio made up of 60% US equities, 20% US Treasury bonds, 10% US corporate bonds and 10% in 3 month US T-bills, rebalanced to those weights every month, would have generated a real return after inflation of 5.1%[12] from 1926 to the present[13] (yes, including the 80% drop in US equity prices from September 1929 to June 1932) and 6.0% from 1975 to the present,

2) *Diversification not only reduces risk but increases returns as well.* From 1926 to the present, a portfolio that had 30% in US equities, 20% in UK equities and 10% in Japanese equities, 20% in US T-bonds, 10% in US corporate bonds and 10% in cash returned 0.1%[14] per annum more than the US only portfolio above and had less risk, despite the fact that Japanese equities lost 98% of their real value through WWII. From 1975 to the present, a more heavily diversified portfolio showed the benefits of diversification more powerfully by outperforming a US-centric portfolio by 0.8% per year over the same period, also with less risk[15], and,
3) *A disciplined approach to portfolio management that periodically rebalances to fixed weights adds to return and increases the quality of those returns*.

The main results that were specific and central in our research are the following two points concerning value and momentum:

1) *Following a strategy of modestly changing exposure to assets as they appear more cheaply or expensively valued adds return of about 1%[16] per annum with our particular set of parameters, and increases the quality of returns by adding more to return than to risk*, and,

2) *Giving a moderate weight to trends increases return[17] and decreases risk quite dramatically.* We looked at only one form of this type of strategy, which involved owning more of an

[12] This return is before transactions costs of rebalancing, fees and taxes. The monthly rebalancing of this strategy would have resulted in a 20% average annual turnover over the period. If the transactions costs on average were ½% per transaction, this would have reduced 0.1% from the 5.1% annual real returns. Today, the transactions costs on these broad asset classes are less than 0.1%. Index fund fees range from 0.5% down to under 0.1% per annum.
[13] Unless otherwise indicated, "present" means to the end of February 2010.
[14] Again, before transactions costs. Turnover here was 30% per annum, or 50% more than in the US portfolio.
[15] Less risk measured in most ways, with the exception that it had a bigger 1 year worst case loss.
[16] Before accounting for almost 100% annual portfolio turnover, compared to 30% for the baseline fixed weight, monthly rebalanced portfolio.
[17] And, as expected, increases portfolio turnover even further, to 150-200% per annum. Given the low transactions costs involved in moving among the major asset classes, even this relatively high degree of portfolio turnover does not result in dramatic diminution of return.

asset when its price was above[18] its average price over the past year, and owning less when its price was below. We will try to explain why it makes sense that this strategy overlay decreases portfolio risk. We will also try to explain why it has increased return in the past, and why we believe it is likely to do so in the future. Many of the explanations of why momentum investing or trend following increases return are in the domain of behavioral economics and rely on explanations that explicitly take into account the fact that markets are made up of people, with all our wonderfully human characteristics.

The essence of this approach is to attempt to identify and avoid cases of extreme overvaluation in asset prices, while providing comfort in holding or buying risky assets when the returns are attractive. Our findings show that superior returns are available to investors using basic valuation metrics and cost efficient strategies to allocate capital among different sources of return across time periods and economic climates. We believe the returns a disciplined investor would have earned following the simple and intuitively appealing investment strategy based on diversification, value and momentum challenges the view that expert active management and alternative investments are preferable to what can be achieved in the public marketplace by investors that are willing to allocate a moderate time commitment to their investment portfolios.

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[18] Above not only the inflation adjusted average price over the past year, but also a small margin above that to take account of the fact that in general prices have drifted up and we didn't want the momentum overlay to have too significant of a long bias.

The tables below give more detailed information from these historical studies, including results for the additional sub-periods of 1990-2010 and 2000-2010[19].

		Real return	Return/Risk	Worst 1 year	Worst 5 year (unannualized)	Worst 10 year (unannualized)	Annual % portfolio turnover (round trip)	Average Weight in Equities
Inflation adjusted returns from 1926								
Inflation- US CPI	3.0%							
US equities		6.4%		-64%	-52%	-45%		
UK equities in $s		5.9%		-61%	-60%	-47%		
Japanese equities in $s		3.4%		-84%	-98%	-98%		
US Treasury 10 year notes		2.2%		-23%	-37%	-42%		
US corporate bonds		3.1%		-25%	-38%	-36%		
US 3-month T-bills		0.8%		-16%	-28%	-42%		
US portfolio with benchmark weights of 60% equities, 20% US T-bonds, 10% US corporate bonds and 10% cash								
Balanced portfolio Buy & Hold (BBH)		**5.1%**	0.42	-42%	-28%	-31%	10%	60.0%
BBH plus Value overlay		**6.0%**	0.45	-49%	-31%	-30%	31%	60.0%
BBH plus Value and Momentum overlay		**6.9%**	0.56	-40%	-21%	-19%	65%	59.7%
More diversified portfolio with benchmark weights of 30% US equities, 20% UK equities, 10% Japanese equities, 20% US T-bonds, 10% US corporate bonds and 10% cash								
Balanced portfolio Buy & Hold (BBH)		**5.2%**	0.54	-35%	-21%	-18%	14%	60.0%
BBH plus Value overlay		**6.1%**	0.54	-44%	-32%	-25%	36%	65.6%
BBH plus Value and Momentum overlay		**7.3%**	0.74	-32%	-25%	-9%	86%	62.4%
Inflation adjusted returns from 1975								
Inflation- US CPI	4.1%							
US equities		7.0%		-43%	-38%	-45%		
US value equities		7.5%		-46%	-38%	-42%		
UK equities in $s		9.4%		-52%	-60%	-35%		
UK value equities in $s		10.3%		-54%	-42%	-39%		
Japanese equities in $s		4.5%		-45%	-55%	-62%		
Japanese value equities in $s		7.7%		-41%	-43%	-48%		
EM equities in $s		9.3%		-58%	-48%	-24%		
Global equities MSCI market capitalization weighted		6.8%	0.45	-48%	-33%	-34%		
Global equities fixed weights (avg GDP weights over period)		8.0%	0.53	-49%	-32%	-16%	15%	
Real estate		7.7%		-56%	-47%	5%		
Commodities		2.2%		-60%	-39%	-27%		
US Treasury 10 year notes		4.1%		-23%	-37%	-42%		
US corporate bonds		5.2%		-25%	-38%	-36%		
US 3-month T-bills		1.6%		-3%	-8%	-10%		
US portfolio with benchmark weights of 60% equities, 20% US T-bonds, 10% US corporate bonds and 10% cash								
Balanced portfolio Buy & Hold (BBH)		**6.0%**	0.60	-27%	-21%	-21%	9%	60.0%
BBH plus Value overlay		**6.5%**	0.65	-30%	-24%	-15%	34%	54.1%
BBH plus Value and Momentum overlay		**6.9%**	0.73	-20%	-16%	2%	73%	53.9%
More diversified portfolio with benchmark weights of 55% global equities, 10% US T-bonds, 5% US corporate bonds, 10% real estate, 10% commodities and 10% cash								
Balanced portfolio Buy & Hold (BBH)		**7.0%**	0.69	-40%	-18%	8%	15%	55.0%
BBH plus Value overlay		**8.2%**	0.84	-35%	-18%	21%	66%	48.6%
BBH plus Value and Momentum overlay		**9.7%**	1.06	-23%	2%	73%	118%	48.7%

[19] All historical returns in this paper are geometric returns, not arithmetic averages of the annual returns. Also, return/risk figures take the realized real return and divide by the samples standard deviation- as such, these should not be viewed as Sharpe Ratio's, which take the excess return over the risk free rate as the numerator.

		Real return	Return/Risk	Worst 1 year	Worst 5 year (unannualized)	Worst 10 year (unannualized)	Annual % portfolio turnover (round trip)
Inflation adjusted returns from 1990							
Inflation- US CPI	2.7%						
US equities		5.4%		-43%	-38%	-45%	
UK equities in $s		4.4%		-52%	-41%	-35%	
Japanese equities in $s		-4.6%		-45%	-55%	-62%	
EM equities in $s		7.3%		-58%	-48%	-24%	
Global equities MSCI market capitalization weighted		3.1%					
Global equities fixed weights (avg GDP weights over period)		4.8%					
Real estate		6.1%		-56%	-47%	5%	
Commodities		1.5%		-60%	-39%	-10%	
Hedge funds (HFR all funds index)		9.1%		-21%	1%		
Hedge funds (HFR all fund of funds index)		5.2%		-22%	-6%		
US Treasury 10 year notes		4.4%		-13%	-2%	28%	
US corporate bonds		5.4%		-21%	-17%	7%	
US 3-month T-bills		1.1%		-3%	-3%	2%	
US portfolio with benchmark weights of 60% equities, 20% US T-bonds, 10% US corporate bonds and 10% cash							
Balanced portfolio Buy & Hold (BBH)		**5.1%**		-27%	-20%	-21%	
BBH plus Value overlay		**4.8%**		-30%	-24%	-15%	
BBH plus Value and Momentum overlay		**5.9%**		-20%	-15%	2%	
More diversified portfolio with benchmark weights of 55% global equities, 10% US T-bonds, 5% US corporate bonds, 10% real estate, 10% commodities and 10% cash							
Balanced portfolio Buy & Hold (BBH)		**5.0%**		-40%	-18%	8%	
BBH plus Value overlay		**5.9%**		-35%	-18%	21%	
BBH plus Value and Momentum overlay		**8.1%**		-23%	2%	73%	118%
Inflation adjusted returns from 2000							
Inflation- US CPI	2.5%						
US equities		-3.7%		-43%	-38%		
UK equities in $s		-1.8%		-52%	-41%		
Japanese equities in $s		-5.6%		-40%	-46%		
EM equities in $s		6.7%		-58%	-48%		
Global equities MSCI market capitalization weighted		-1.9%					
Global equities fixed weights (avg GDP weights over period)		-0.1%					
Real estate		7.2%		-56%	-47%		
Commodities		2.1%		-60%	-39%		
Hedge funds (HFR all funds index)		3.7%		-21%	1%		
Hedge funds (HFR all fund of funds index)		1.3%		-22%	-6%		
HFR Hedge fund index		1.8%		-25%	-20%		
US Treasury 10 year notes		3.9%		-11%	-2%		
US corporate bonds		5.0%		-21%	-17%		
US 3-month T-bills		0.2%		-3%	-3%		
US portfolio with benchmark weights of 60% equities, 20% US T-bonds, 10% US corporate bonds and 10% cash							
Balanced portfolio Buy & Hold (BBH)		**-0.6%**					
BBH plus Value overlay		**1.3%**					
BBH plus Value and Momentum overlay		**2.6%**					
More diversified portfolio with benchmark weights of 55% global equities, 10% US T-bonds, 5% US corporate bonds, 10% real estate, 10% commodities and 10% cash							
Balanced portfolio Buy & Hold (BBH)		**2.7%**		-40%	-18%	n/a	
BBH plus Value overlay		**4.6%**		-35%	-18%	n/a	
BBH plus Value and Momentum overlay		**7.5%**		-23%	2%	n/a	113%

[20]

[20] We made many marginal changes to the structure and parameterization of our framework to check the stability of the historical returns presented above. In general, we found the results fairly robust. In the end, of course, this is a purely qualitative statement, as what we consider modest or marginal changes others might call very small changes indeed. Furthermore, we recognize that the quality of the data that we have is not very high. There are many instances

2) Asset classes used in the studies

We chose the asset classes for this study based on the following criteria:

- Big enough to feel like an asset class and to carry risk that isn't easy for investors in aggregate to diversify (aka systematic risk)
- Assets available for investment through low cost, liquid vehicles where ordinary investors could get exposure and have low transactions costs in rebalancing. These include index funds and ETFs[21]
- Indices where we could find reasonably accurate historical data, including information not only on historical prices, but also total returns including dividends and also some valuation metrics, such as earnings

The following asset classes were used in the four historical studies:

1) End of January 1926 to the end of February 2010 (~84 years)
    a. "US Only" portfolio
        - US equities
        - US 10 year T-notes
        - US investment grade corporate bonds
        - US 3 month T-bills
    b. "Three Country" portfolio, as in 1a above plus
        - UK equities[22],
        - Japanese equities
2) End of December 1974 to February 2010 (~35 years)
    a. US Only: as in 1926 study above
    b. "Broad Portfolio": as in 2a above plus
        - US value equities
        - UK broad and value equities
        - European broad and value equities
        - Japanese broad and value equities
        - Pacific developed non-Japanese broad and value equities
        - Canadian broad and value equities
        - Emerging market equities
        - US Real estate (REIT index)
        - Commodities (GSCI index)

The term "value equities" refers to that sector of the broad equity market having lower price-to-book and price-earnings ratios than the average of the broad market. Historically, the value sector of the market has generated higher returns than the growth sector (its complement) and therefore

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of data points that look unlikely to be correct and other cases where there are clear problems in our data not being synchronously gathered at the end of each month. We believe that in general, this leads to a mild overstatement of the buy and hold returns, a mild but more significant overstatement of the incremental returns from scaling based on value, and probably has a mild effect of understating the incremental returns added by the momentum overlay.

[21] And financial futures for more sophisticated investors.

[22] Returns in $US for all non-US equity markets. Research into foreign currency hedging is work in progress.

the broad market as well. Various risk premium arguments have been put forward to explain this extra return.[23] For instance, many observers view this as compensation for the risk that value equities will do worse in a severe economic downturn. Fortunately, there have not been that many truly severe economic downturns to be able to test this theory out. During the bear market from late 1929 to mid 1932 when the broad US equity market lost 80% of its real value, value equities went down by well over 90%. However, in the recent credit crisis from mid 2007 to mid 2009, US value equities only slightly under-performed the broad market's performance.[24]

There is a body of empirical research that leads us to feel that value equities[25] warrant inclusion as an asset class in a balanced portfolio.

3) Defining the baseline portfolio

We needed to reach some decision on what would constitute a "balanced", baseline portfolio for the purpose of these historical studies. As the main objective of this research note is to investigate value and momentum overlays to a diversified constant weight buy and hold portfolio[26], we decided to place the fuller discussion of buy and hold strategies into Appendix 1 to allow the reader to proceed more directly to the heart of the matter.

Our choice of the balanced, buy and hold portfolios that form the starting point of our research was heavily influenced by what seemed to be the most common weightings among moderately risk-averse investors.[27] We settled on baseline portfolios consisting of about 60% in equities, with the rest split between the other available assets, including 10% in cash. As the breadth of data available to us declined the further back in time we probed, we decided to construct several different baseline portfolios, which we describe below.

These portfolios, which we call the "Balanced Buy and Hold" or "BBH" portfolios, consisted of one or more publicly traded equity indices, US government and investment grade bonds, US T-bills, and, in the most diversified portfolio, indices of real estate and commodities. In each case, we started by examining the historical returns of those portfolios rebalanced back to fixed weights at the end of each month[28]. The simplest of the three portfolios we constructed consisted of:

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[23] Fama and French did some of the earliest and most referenced work in this field.

[24] From June 30, 2007 to June 30, 2009 the US broad market had an excess return of -21% versus the Value sector's -24.5% annual return.

[25] and small cap equities as well, although we haven't included them in the study.

[26] From here on, when we refer to "buy and hold" portfolios, we mean a portfolio with constant weights assigned to holdings of the constituent investments, where the portfolio is rebalanced back to those fixed weights on a monthly basis. We hope that referring to them simply as buy and hold portfolios does not cause confusion.

[27] John C. Bogle, founder of the Vanguard Group of mutual funds that manages $1.4Trillion of assets: "I believe it is prudent to own a balanced index fund with 60 per cent in the stock market and 40% in the bond market. Then forget it. Don't worry. Stay the course. An easy way of thinking about asset allocation is to make your bond allocation equal to your age. So, at my age, 81, I have about 80% of my money in bonds and the rest in equities. I am very happy being conservative." (interview for the FT November 6th, 2010).

[28] As mentioned elsewhere, it would be very interesting to perform all of the analysis in this paper with quarterly or annual rebalancing frequencies. As described in a separate footnote, we have looked at other rebalancing frequencies in the case of the 1975-2010 broadly diversified portfolio.

- 60% US equities
- 20% US 10 year Treasury notes
- 10% US 10 year investment grade corporate bonds
- 10% US 3 month T-bills

We used two other more diversified portfolios to explore the effects of diversification relative to the exclusively US portfolio above. The two other portfolios were:

1926-2010: Three country			1975-2010: Broadly diversified	
30%	US Equities			Equities
20%	UK Equities		10.0%	US broad
10%	Japanese Equities		10.0%	US value
20%	US Investment Grade Corporate bonds		4.5%	Europe broad
10%	US 10 year Treasury note		4.5%	Europe value
10%	US 3 month T-bills		2.5%	UK broad
			2.5%	UK value
100%	Total		3.5%	Japan broad
			3.5%	Japan value
			2.0%	Pacific x-Japan broad
			2.0%	Pacific x-Japan value
			1.0%	Canada broad
			1.0%	Canada value
			8.0%	Emerging Market broad
			10.0%	US 10 year Treasury note
			5.0%	US Investment Grade Corporate bonds
			10.0%	Real Estate (REITs)
			10.0%	Commodities (GSCI)
			10.0%	US 3 month T-bills
			100%	Total

4) The mechanics: A step by step guide of how these strategies were constructed for the historical studies

Here is how the historical studies described in this paper were implemented:

1) *Chose a benchmark, base weighting of assets to form the baseline portfolio.* As described above, we used three baseline portfolios. In the case of the pure buy and hold strategy, rebalance back to the baseline weights at the end of each month.
2) *Chose the valuation measures and assign an attractiveness measure, on a scale from 0 to 10, to those valuation metrics.* For instance, when looking at equity markets, we assign a 0 (least attractive) to the market when the P/E[29] is greater than 32, a 10 (most attractive) when the P/E is less than 8, and a 5 when the P/E is between 16 and 18. We use a simple valuation metric, discussed in more detail in section 4 below for each asset class under consideration in our portfolio.

[29] The Shiller P/E as described earlier.

3) *Determine how much to increase or decrease exposure relative to baseline given attractiveness scale of 0 to 10.* In all of our studies, we went with a very simple scheme of increasing or decreasing baseline exposure by 2/3rd when the attractiveness measure was at its most extreme. For attractiveness measures in between, we adjusted the weighting of the asset in question by proportionately less than the 2/3's maximum adjustment.[30]
4) *Chose a simple measure for momentum.* We used the difference between today's price and the average inflation adjusted price over the past year as our measure of momentum. [31]
5) *As with value, map the momentum into a desired change to the baseline exposures.* Here we decided to decrease by 1/3rd when momentum is negative and to increase by 1/3rd when positive[32]. It is interesting to note here that we vary portfolio weights less based on momentum than we do based on valuation changes, because, despite the fact that momentum has been a stronger signal historically than value, we feel more comfortable with the view that value will be more likely to persist going forward than momentum.
6) *Additionally, and importantly, we did not allow the portfolio to have any leverage, so when the desired baseline weights, after making the desired adjustments for value and momentum, added up to more than 100%, we scaled all the desired weights down by the sum so that the actual exposures added up to 100%[33].*

This six step process is all that we did in order to arrive at the historical return results we present in this paper. We chose to go through this rebalancing process on a monthly basis.[34] For the "Buy and Hold" strategy, we stopped at step 1, rebalancing back to those weights at the end of each month. For the full strategy of "Buy and Hold with Value and Momentum" we used all 6 steps, performing steps 2 through 6 at the end of each month, while for the "Buy and Hold and Value" strategy we left out steps 4 and 5.[35]

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[30] In fact, the scaling winds up being asymmetric in that it is easy to reduce exposures, but there is not much room, due to the small amount of cash in the baseline portfolio and no-leverage constraint, to increase positions when risky assets in general are attractive.

[31] Because all assets are expected in general to earn more than the rate of inflation, we actually defined positive momentum to be when today's price is a small margin above the last year's average price after inflation. Also, we recognize that in practice, having a no- trade range might be a prudent thing to do in order to somewhat reduce transactions costs, but for simplicity we left it out of our base historical runs. In fact, it made little difference to the historical returns or portfolio turnover numbers.

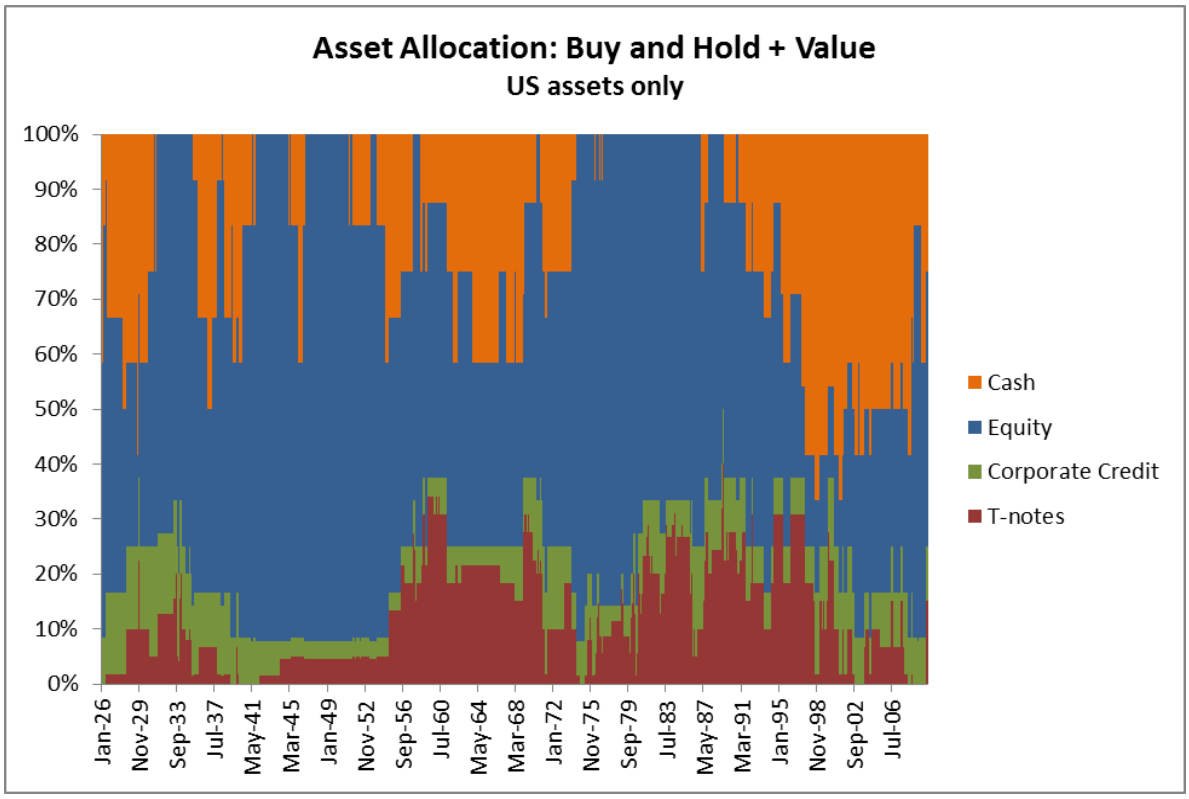
[32] There are other ways of getting roughly the same effect with lower portfolio turnover, and greater synergy. For example, if portfolio scaling based on value were done slowly, with a lag, in effect that would be incorporating momentum into the portfolio strategy. This can be seen by considering a case where an asset has gone down in price and based on value, the portfolio would be adding to that asset. However, at that instant, momentum would likely be negative, and so waiting for some time for the moving average to drift down towards the new price would be a form of momentum overlay. We intend to explore that type of integration of value and momentum in future research.

[33] We also did not want the portfolio to have outright short positions in any asset class.

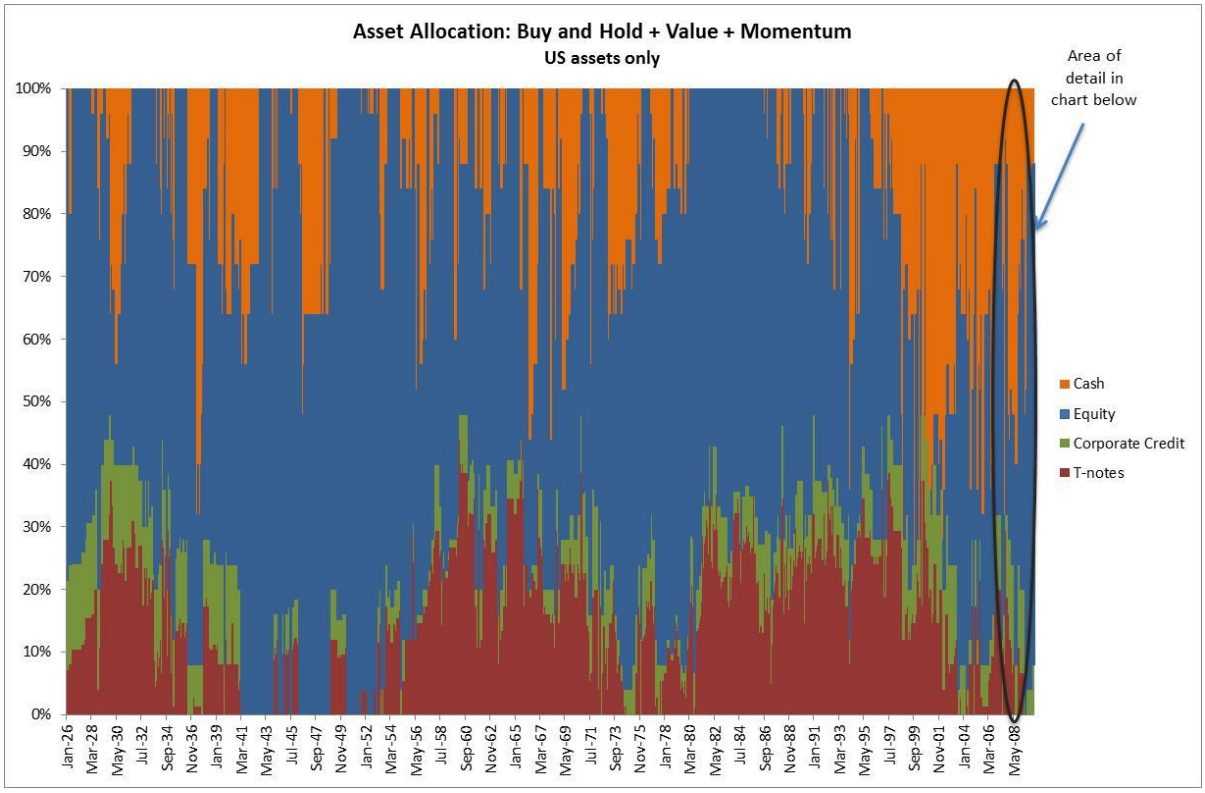
[34] After we had completed most of this study, and 3 months after writing up the first draft, we looked at the effect of longer rebalancing frequencies on historical returns. In particular, we looked at the impact on the 1975-2010 broadly diversified portfolio returns. What we found was what we broadly expected, which is that rebalancing on a less frequent basis helps the value strategy and hurts the momentum component of returns. One way of interpreting this result is that rebalancing to value signals slowly is a form of putting a trend following overlay onto the strategy. We found that annual rebalancing added about 0.25% pa to the returns of the value strategy, and that quarterly rebalancing added about ½ that. By contrast, annual rebalancing clipped a whopping 0.8% from the value and momentum combined strategy, and quarterly rebalancing subtracted about 0.17% from their returns.

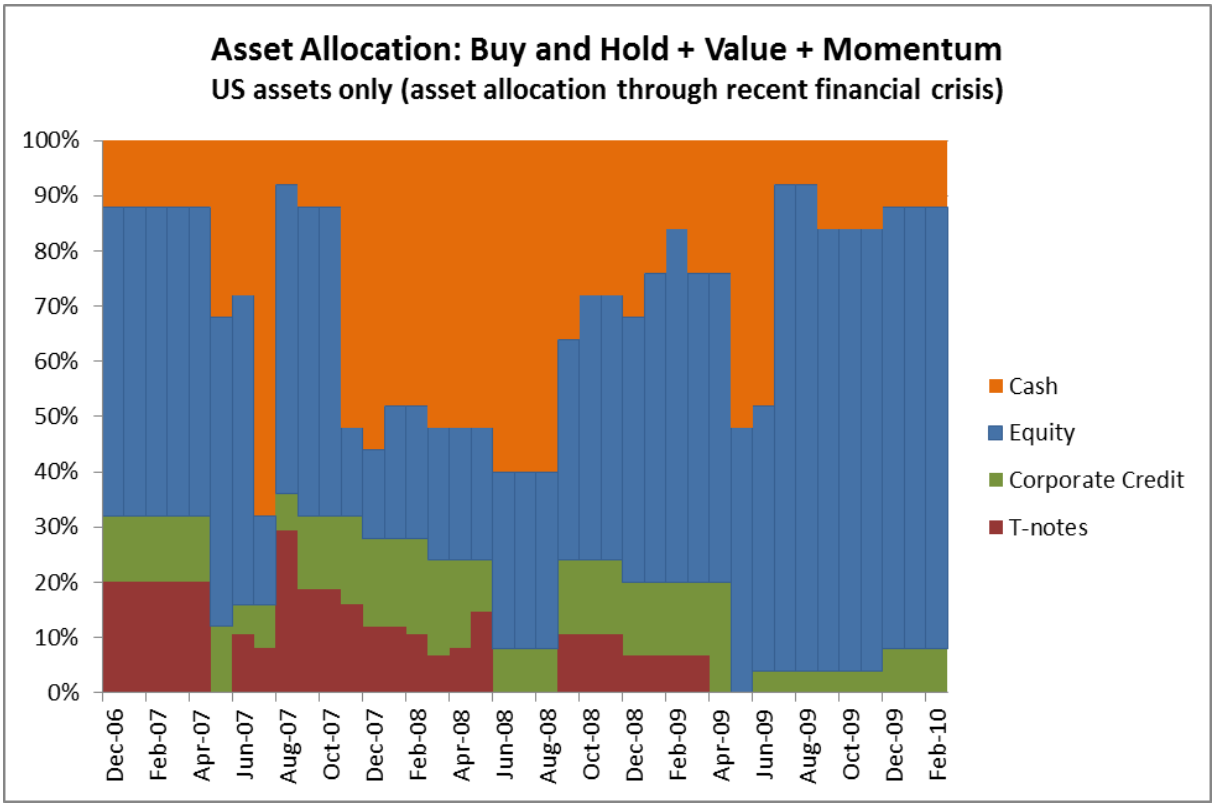
[35] Another variation we explored was to only change the weighting based on value for changes in the attractiveness index of more than 1 step, in effect imposing a trading buffer on the strategy. The result of this was to slightly decrease portfolio

The charts below show how the asset allocation changed over time for a portfolio invested solely in US assets from 1926. The first chart shows the allocation when only the value criteria are applied, and the second chart shows the asset allocation when the momentum overlay is applied on top of the value oriented portfolio. The third chart shows greater detail of the rebalancing through the recent financial crisis from the start of 2007 to early 2010.



turnover and to add about 15% to the increase in returns attributable to the value based rebalancing strategy (ie. about 15bp of extra return in the value strategy).


Asset Allocation: Buy and Hold + Value + Momentum
US assets only
Area of detail in chart below
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Jan-26
Mar-28
May-30
Jul-32
Sep-34
Nov-36
Jan-39
Mar-41
May-43
Jul-45
Sep-47
Nov-49
Jan-52
Mar-54
May-56
Jul-58
Sep-60
Nov-62
Jan-65
Mar-67
May-69
Jul-71
Sep-73
Nov-75
Jan-78
Mar-80
May-82
Jul-84
Sep-86
Nov-88
Jan-91
Mar-93
May-95
Jul-97
Sep-99
Nov-01
Jan-04
Mar-06
May-08
Cash
Equity
Corporate Credit
T-notes


Asset Allocation: Buy and Hold + Value + Momentum
US assets only (asset allocation through recent financial crisis)
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-06
Feb-07
Apr-07
Jun-07
Aug-07
Oct-07
Dec-07
Feb-08
Apr-08
Jun-08
Aug-08
Oct-08
Dec-08
Feb-09
Apr-09
Jun-09
Aug-09
Oct-09
Dec-09
Feb-10
Cash
Equity
Corporate Credit
T-notes

5) Simple measures of value

For each of our asset classes, the simple valuation metric that we have used is as follows:

1) *Equity markets*: P/E where P is the current stock market index level, and where E is the average of the past 10 years of inflation adjusted corporate earnings on that index[36]
2) *US 10 year Treasury notes*[37]: the expected 10 year real return, which is the difference between the next 10 years' forecasted inflation and the current yield of the 10 year Treasury note[38]. There is not a more direct measure of the long term expected real return than this; it is in a sense the definition of the long term expected real return
3) *US Investment Grade Corporate bonds*: the yield spread, which is the difference in yield between the index of corporate bonds and the yield on a maturity matched portfolio of US treasury bonds. We are using a relative measure (i.e. the spread to Treasuries) because we want to determine how much of the total interest rate risk we want in the portfolio, as determined by the metric on US 10 year notes, is to be in the form of corporate bonds.
4) *US Real Estate (REIT) Index*: the dividend yield based on the prior year's dividends on the REIT index
5) *Commodities (GSCI index):* here is the sole case where we use two metrics to give us our indicator of expected returns. We give equal weight to measures of: i) today's price relative to the average inflation adjusted price over the past 10 years, and ii) the difference between today's spot commodity prices and the prices of those commodities for future delivery as indicated by futures market pricing.[39]
6) *Value equities*: in each market[40] we look at the subset of the broad market with the cheapest valuation measures, known as the "value" segment of the market. MSCI, the data vendor, splits each broad equity index into Value and Growth sectors and reports total returns for each. Our measure of value in the analysis of Value stocks (yes, an unfortunate and confusing naming convention) is simply the recent

[36] The reader may question why interest rates do not come in to the evaluation of equity market cheapness. The reason is that we are trying to make an investment decision about how much to invest in equities and not how much to go long equities versus short bonds. The metric has to be consistent with the position it is being used to determine. In a scenario where equities are expensive when viewed in isolation but cheap relative to bonds, and bonds themselves are offering low returns, the portfolio our analysis would indicate would have low exposures to equities and bonds.

[37] We also looked at three other measures which all added to predictability but for simplicity we left them out of this study. They were: 1) current level of interest rate forward (20 year 5 years forward) compared to its average over the past 10 years, 2) current short term rate compared with average inflation over the past 3 years, and 3) the current level of the short term rate compared to its level 6 months ago.

[38] We would have preferred to use the real yield on TIPs for this, but US inflation indexed bonds have existed for less than 15 years.

[39] This is known as contango when futures prices are above spot prices and backwardation when futures prices are below spot prices. We use the next 3 months difference between the return of the GSCI total return index (futures plus cash) versus the GSCI spot index (invested in physicals without storage cost) to estimate the amount of backwardation that would have been expected (with perfect foresight) each period. Unfortunately, we may have done this in a manner that was distorted by seasonality in some of the commodity prices, and will revisit this shortly.

[40] Except for emerging markets.

historical outperformance or underperformance of value stocks versus the broad equity index over the most recent three years[41]. As with corporate bonds, above, we are using a relative measure of cheapness for Value equities because, in the construction of the portfolio, we are going to use this metric to decide how much of our total equity exposure in a given market is to be held in the form of the Value sector.

We hope that in each of these cases, it is clear to the reader why, and in which direction, these measures are related to the assessment of whether a particular asset is cheaply or richly priced. In the table below we show the mapping from each measure to the cardinal values of 0 to 10.

Asset -->	Equities	US 10yr T-note	US corp bonds	US REITs	Commodities	Commodities	Value equities
Valuation Metric -->	Shiller P/E (past 10yr real avg)	expected 10 year real rate (10 yr nom yld - expected 10 yr infl)	spread to Treasury bonds	dividend yield	current price / 10 year real avg price	backwardation: 3 month forward	annual return of Value vs Broad equity index over past 3 years
0	> 32	< 1%	< 0.75%	< 5%	> 1.625	< -7%	> 5.25%
1	28 to 32	1.0% to 1.5%	0.75% to 1%	5% to 5.5%	1.5 to 1.625	-7% to -5%	4% to 5.25%
2	24 to 28	1.5% to 2.0%	1.0% to 1.3%	5.5% to 6%	1.375 to 1.5	-5% to -3%	2.75% to 4%
3	20 to 24	2.0% to 2.5%	1.3% to 1.5%	6% to 6.5%	1.25 to 1.5	-3% to -1%	1.5% to 2.75%
4	18 to 20	2.5% to 2.8%	1.5% to 1.7%	6.5% to 7%	1.2 to 1.3	-1.0% to 0.3%	0.8% to 1.5%
5	16 to 18	2.8% to 3.2%	1.7% to 1.8%	7% to 7.5%	1.1 to 1.2	0.3% to 1.7%	0.2% to 0.8%
6	14 to 16	3.2% to 3.5%	1.8% to 2.0%	7.5% to 8%	1.0 to 1.1	1.7% to 3.0%	-0.5% to 0.2%
7	12 to 14	3.5% to 4.3%	2.0% to 2.5%	8% to 9%	0.9 to 1.0	3.0% to 5.0%	-1.8% to -0.5%
8	10 to 12	4.3% to 5.0%	2.5% to 3.0%	9% to 10%	0.7 to 0.8	5.0% to 7.0%	-3.0% to -1.8%
9	8 to 10	5.0% to 5.8%	3.0% to 3.5%	10% to 11%	0.8 to 0.9	7.0% to 9.0%	-4.3% to -3.0%
10	< 8	> 5.75%	> 3.5%	> 11%	< 0.8	> 9%	< -4.25%

[42]

For most of these measures, the relationship between the predictor and the return is quite direct when viewed to a very long term horizon[43]. Perhaps the case is most sharply in focus

[41] We also tested two other measures: 1) the ratio of the current price to book value of value stocks compared to the price to book ratio of the broad market, and 2) the ratio of the P/E of the value stocks compared to the P/E of the broad market. These measures added to predictability, but, in the interest of simplicity we decided to leave them out of the analysis we report in this paper.

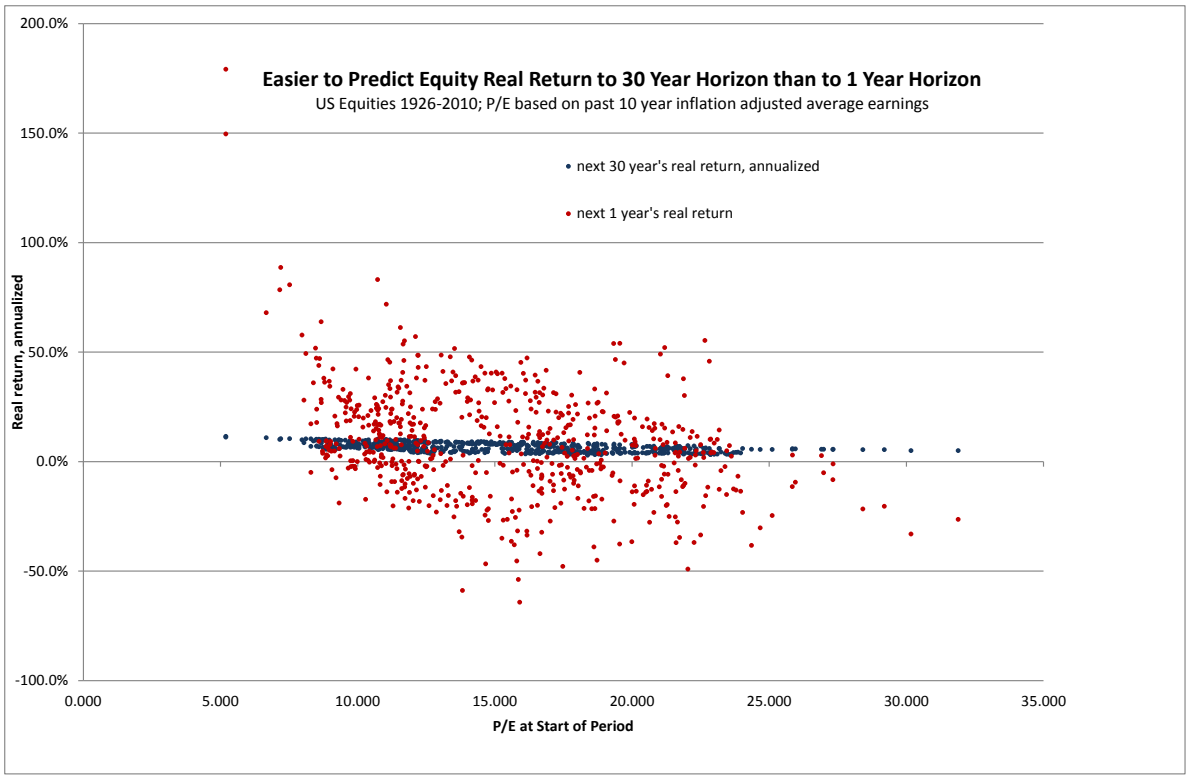
[42] For simplicity, we have used the same P/E cutoff points for all global equity markets. In practice, it would seem more appropriate to use slightly lower or higher P/E's depending on the market, as there are differences in the broad characteristics of each market, as well as accounting practices. When we applied different base P/E's to each market, we found that historically the returns (relative to risks) achieved were slightly more attractive.

[43] Other writers have also made this point, namely, that as the prediction horizon gets longer, the noise to signal ratio in predicting returns in the main asset classes gets lower.

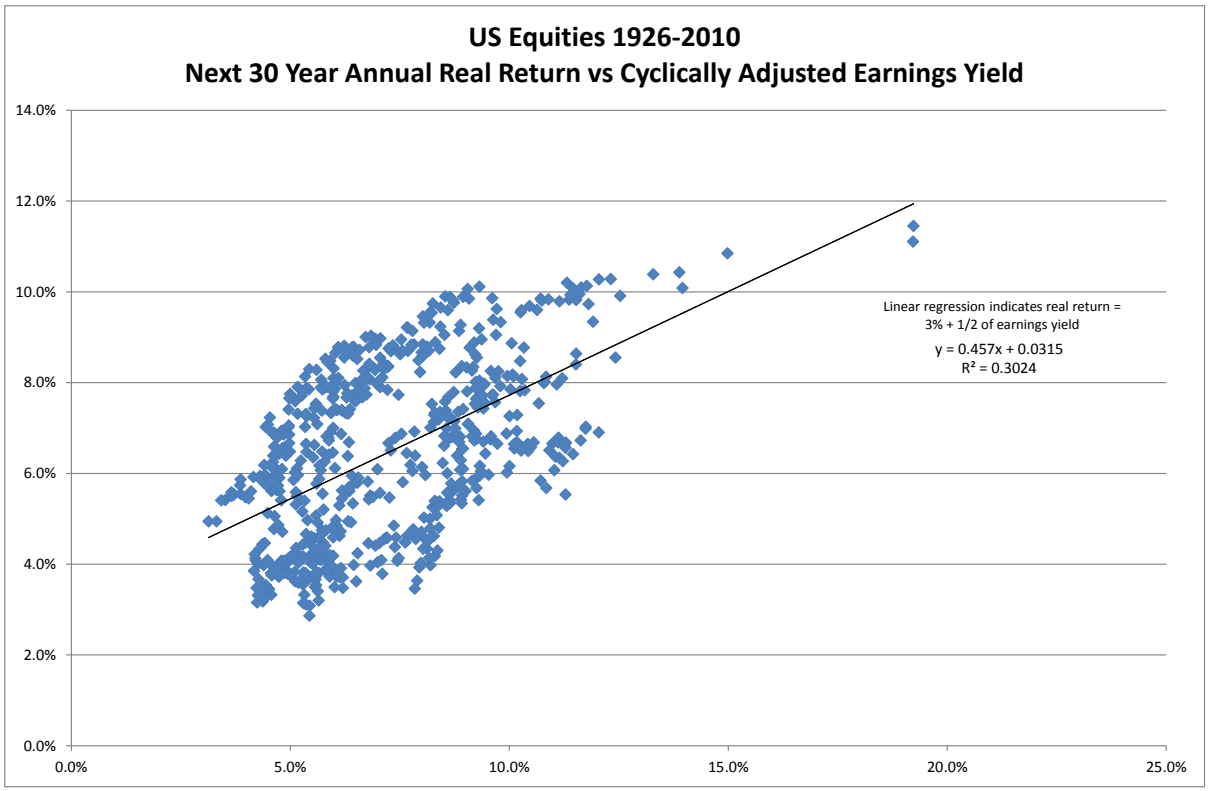
when we look at predicting the long term real return of US 10 year Treasury notes. Our metric is the difference between the yield on the 10 year bond and expected inflation over the next 10 years. To a 10 year horizon, the only risk to that prediction of the 10 year real rate is what inflation turns out to be. That forecast does indeed have quite a bit of uncertainty; from 1975 to the end of 2009 the standard error of the forecast was 1.4%, meaning that roughly 2/3 of the time the forecast of inflation turned out to be within plus or minus 1.4% of the actual outcome for the next 10 years of inflation, and on average the forecast overestimated actual inflation by 1%. On the other hand, trying to guess the move in the 10 year rate over the next 3 months is an even more difficult task, and on average the 10 year rate moved with a standard deviation of 0.9% each quarter, just a bit below the total uncertainty over the whole 10 year period when forecasting inflation.

Similarly, with equities, to a very long term horizon, most of the return an investor will earn will come from dividends[44], and so, if we believe that there is a strong link between long term earnings and dividends (ie. a stable payout ratio) and we believe that earnings and dividends will grow with some relatively stable relationship to long term growth in the economy, then we should expect to see a good linkage between long term equity returns and the amount we have to pay for each dollar of earnings (which is the P/E ratio). True, there are a lot of "ifs" in that relationship, but it turns out that when we look historically over the past 100 years, those relationships do seem to be quite stable. The chart below shows the contrast between trying to forecast the next one year return to the equity market compared with forecasting the next 30 year annual return.

[44] And share buybacks for companies trying to minimize dividends for tax efficiency. In a fuller analysis, the stated dividend rate would need to be adjusted for a number of other similar events, such as issuance of new equity, issuance of stock options to management, cash tenders, etc.

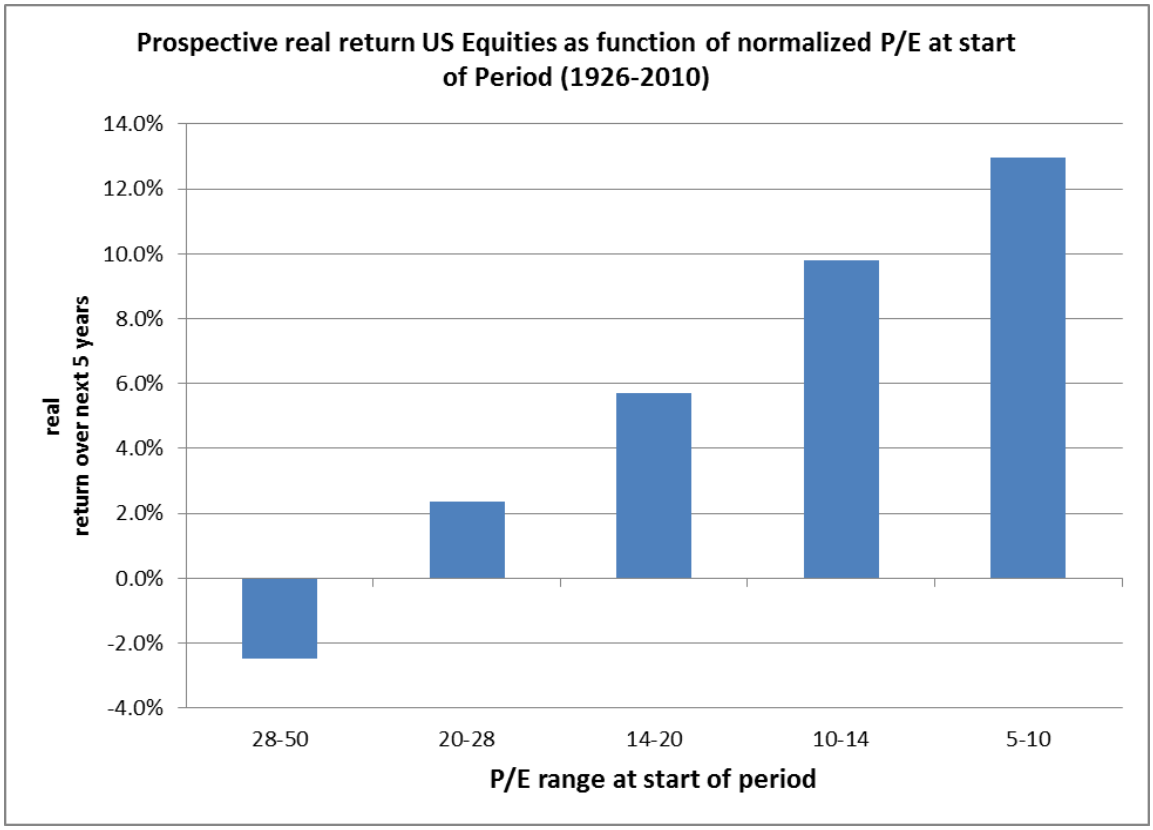


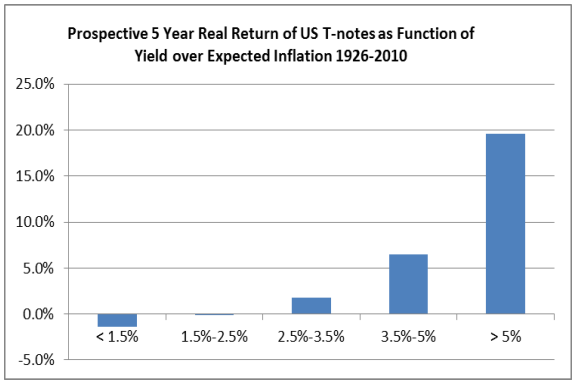
The next chart shows the 30 year prospective return of equities as a function of earnings yield at the start of the period. In effect, it is zooming in on the dark dots from the chart above, ignoring the wider dispersion of one year returns.

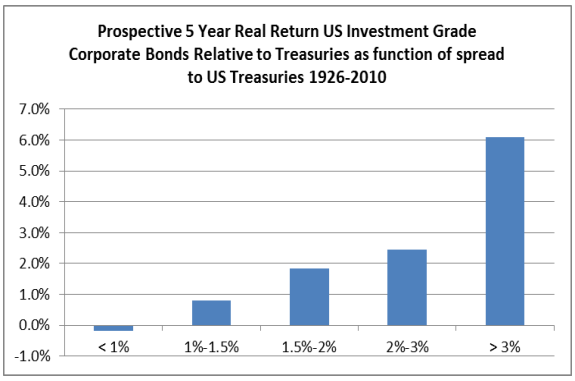


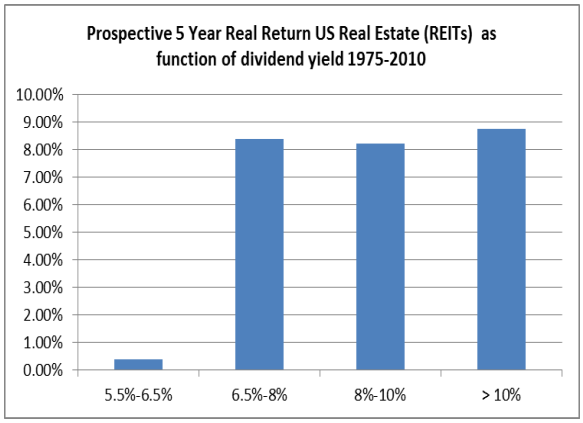
The seven charts below explore the relevance of these long term valuation metrics to a shorter horizon of five years. They demonstrate that we don't have to commit to a 30 year horizon to see a good deal of predictability in returns as a function of simple valuation metrics.[45]

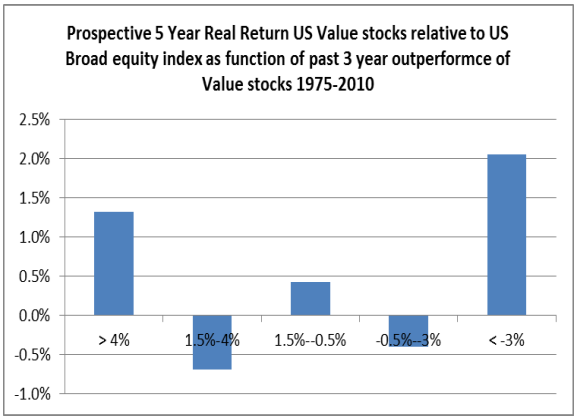
[45] It appears that all the other assets we've included in the study from 1975, namely the other global equity markets and the Value sectors of them, also show on average a similar degree of predictability based on these measures.

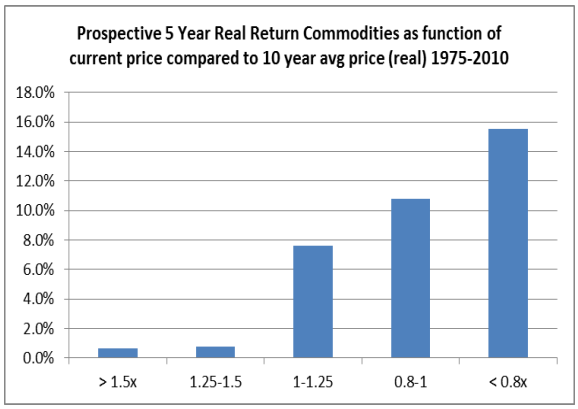

Prospective real return US Equities as function of normalized P/E at start of Period (1926-2010)
14.0%
12.0%
10.0%
8.0%
6.0%
4.0%
2.0%
0.0%
-2.0%
-4.0%
real return over next 5 years
28-50
20-28
14-20
10-14
5-10
P/E range at start of period

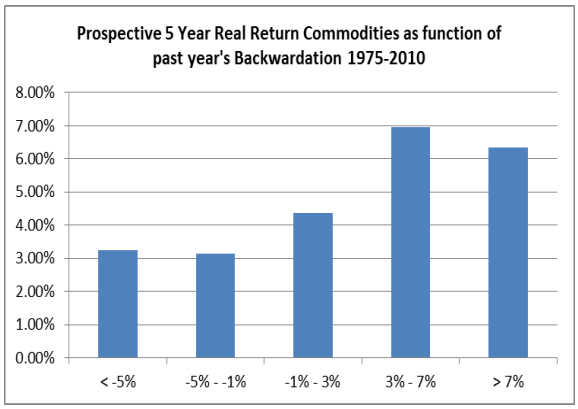

Prospective 5 Year Real Return of US T-notes as Function of Yield over Expected Inflation 1926-2010
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
< 1.5%
1.5%-2.5%
2.5%-3.5%
3.5%-5%
> 5%


Prospective 5 Year Real Return US Investment Grade Corporate Bonds Relative to Treasuries as function of spread to US Treasuries 1926-2010
7.0%
6.0%
5.0%
4.0%
3.0%
2.0%
1.0%
0.0%
-1.0%
< 1%
1%-1.5%
1.5%-2%
2%-3%
> 3%

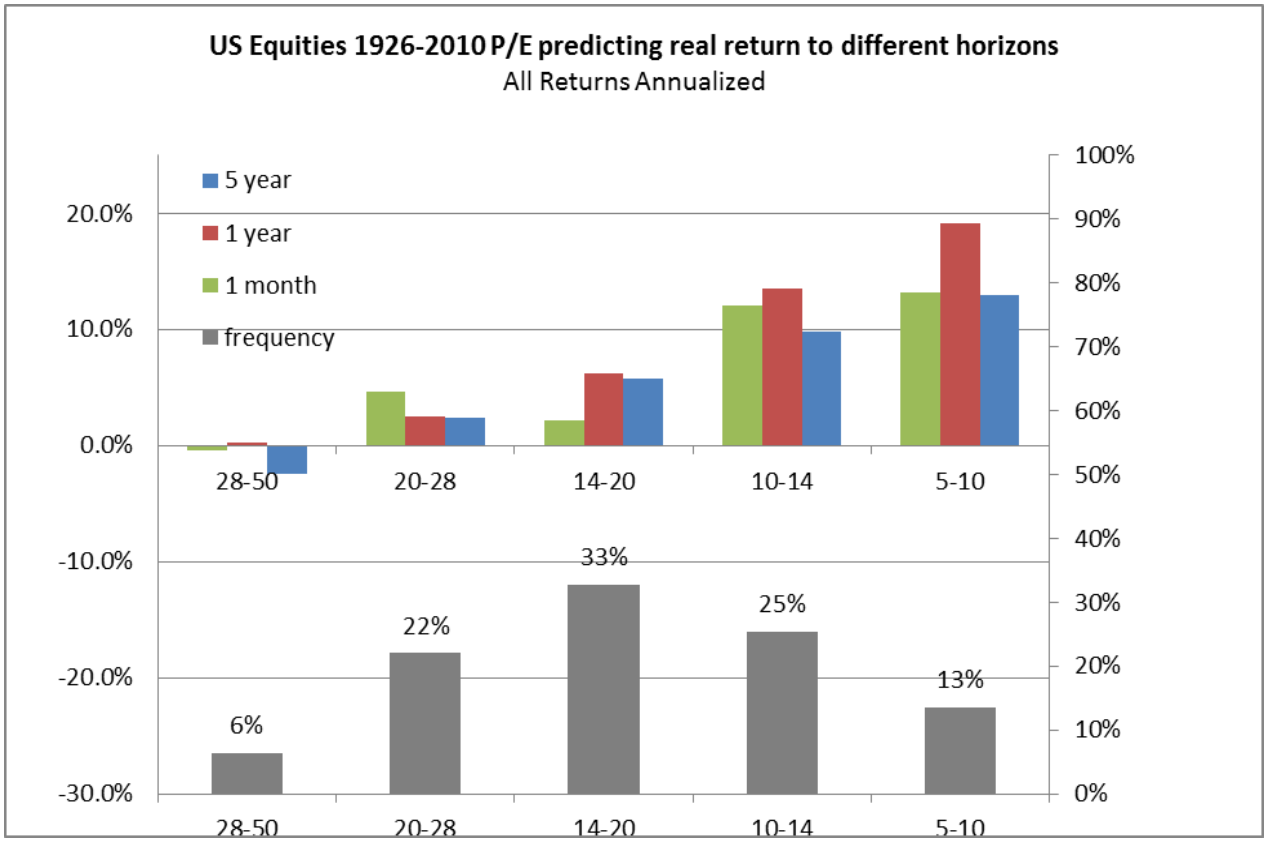

Prospective 5 Year Real Return US Real Estate (REITs) as function of dividend yield 1975-2010
10.00%
9.00%
8.00%
7.00%
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
5.5%-6.5%
6.5%-8%
8%-10%
> 10%


Prospective 5 Year Real Return US Value stocks relative to US Broad equity index as function of past 3 year outperformce of Value stocks 1975-2010
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
-0.5%
-1.0%
> 4%
1.5%-4%
1.5%--0.5%
-0.5%--3%
< -3%





Although all the above charts show the next 5 years of return as a function of the valuation of each asset, it also is the case that these simple valuation metrics have a degree of predictive power over the next year, although with a lower signal to noise ratio.[46] This can be seen in the chart below, which shows the next 5 year, 1 year and 1 month return as a function of P/E.[47] In the bottom panel of the chart is the frequency of months that equities had P/E's in those ranges.



[46] And even the next month, at least in the case of US equities from 1926, as per the chart.

[47] The chart somewhat overstates the power of prediction to shorter horizons in that the dispersion of returns to shorter horizons is also wider and so the signal to noise ratio of the P/E as a predictor is lower to the shorter horizons than to the five year horizon.

Given these charts, it is somewhat surprising that scaling one's investments based on value does not add more to long term returns. In our studies, we find that changing allocations based on value adds only about 1% to long term performance, and perhaps more unexpectedly, it does this without dramatically improving the risk-return balance of the portfolio.

From the chart above, it appears that returns over the next year are approximately 6% above or below average depending on whether the P/E is low or high. If we take the simplest example of a portfolio that has 60% equities and when equities are cheap it scales up to 90% equities and when equities are expensive it cuts back to 30% equities, and 2/3rd of the time equities are either cheap or rich (as was the case from 1926 as shown in the chart above), then we'd estimate that we'd get an extra return of about 1.2%[48] per annum. In the strategy that we analyzed, there is much less room for increasing and decreasing equities, given that we start off with only 10% in cash and we have a leverage constraint that squeezes all our desired asset exposures down when multiple assets are attractive at the same time. On the other hand though, the above example would be something of an underestimate due to the fact that we are applying value based scaling to more than just equities. All in all, the roughly 0.8% increase in returns from scaling according to changes in valuation seems plausible if a bit low. As we would expect, when operated over a wider range of assets, the increase in returns due to value based scaling tends to be higher, although this is not always the case. There is even one case that we studied (US only from 1990-2010) where value based scaling seems to have underperformed the buy and hold performance. As discussed earlier, we should not turn up our noses at the availability of an extra 1% of returns- it makes a big difference over the long run.

We also found that value-based changes in allocation added the most when the starting baseline allocations were the most even. If we started with a baseline allocation of 100% in US equities, then the only thing we could do would be to reduce that allocation when they were expensively valued, we found that the value overlay did not materially improve the risk adjusted performance above baseline. The biggest gains from changing allocations based on valuation were evident in the broadly diversified portfolio which we studied from 1975 to 2010.

Overall we were disappointed that value based scaling did not lower risk more than it did, either in absolute terms or relative to the returns. One explanation for this is that for a given average exposure over time, the least risky way to hold that exposure is to hold it constant over time. For example, it is riskier to own $100 of equities for the first 6 months of every year, and be in T-bills for the other 6 months, than it is to hold $50 of equities for the whole year[49]. It may be that depending on how we view risk, value based investing may be less risky than it appears to be

[48] 6% benefit * 2/3 of the time on 30% more or less equities = 1.2%. This is very much an approximation, and there are non-linearities that come into play which makes it easiest to just compute the strategy returns to determine the size of the effects.
[49] Generally, risk goes up with the square root of time, so holding 2x the position for ½ the time tends to be sqrt(2)=1.41x riskier. The applicability of this effect to value based scaling was pointed out to me by my friend and former colleague Vladimir Ragulin.

based on common risk measures. In the section on risk below, we touch on other possibly more relevant concepts of risk which may make value based investing seem less relatively risky.

6) Momentum

In some ways, investing based on trend following is the antithesis of investing based on value. A leading professional trend following investment manager recently remarked: "Value based investing makes a lot of sense, but it doesn't work; trend following makes little sense, but it delivers". After so many years of observing the markets exhibit trends, we must recognize the possibility that momentum is as much a permanent feature of the market landscape as bubbles and crashes. In fact, many practitioners and theoreticians believe that momentum is an integral part of the process which gives us those booms and busts.[50] The existence of momentum makes value investing difficult, as the term "catching a falling knife" conveys. Likewise, trends don't carry on forever, as eventually asset prices drift back in the direction of fair value. If it weren't for momentum, value investing would be too attractive to exist and likewise, the reversion to fair value (and the excess volatility that goes with it) prevents trend following strategies from being too much of a "free lunch".

There are a number of explanations of why momentum is a fixed feature of financial markets. These explanations generally draw on a body of academic research called "behaviorial finance" and include well documented human behaviorial characteristics such as anchoring, the disposition effect, herding and confirmation bias. In addition, it seems that some elements of market structure, such as the effect of government intervention and VaR based risk management regimes, may also play a role in causing, or allowing, trends to exist.[51] We tend to favor the explanations that are based on feedback loops- what George Soros has explained at length in his books as "reflexivity".

Trends can develop (and with increasing strength) when investors believe, contrary to the legally required warning labels on all investment products, that past returns <u>are</u> indicative of future returns. If investors were not hard wired to think in that way, we wouldn't see almost all investment marketing lead off with the attractive past returns investors have enjoyed, with a footnote to the public health warning at the bottom of the page. This perspective manifests itself everywhere; just flip on the TV to one of the stock market channels and observe market commentators and investors alike referring to asset price moves in the present tense, as in "the market is go**<u>ing</u>** up" or "Microsoft's stock is go**<u>ing</u>** down", as opposed the more accurate use of the past tense.[52]

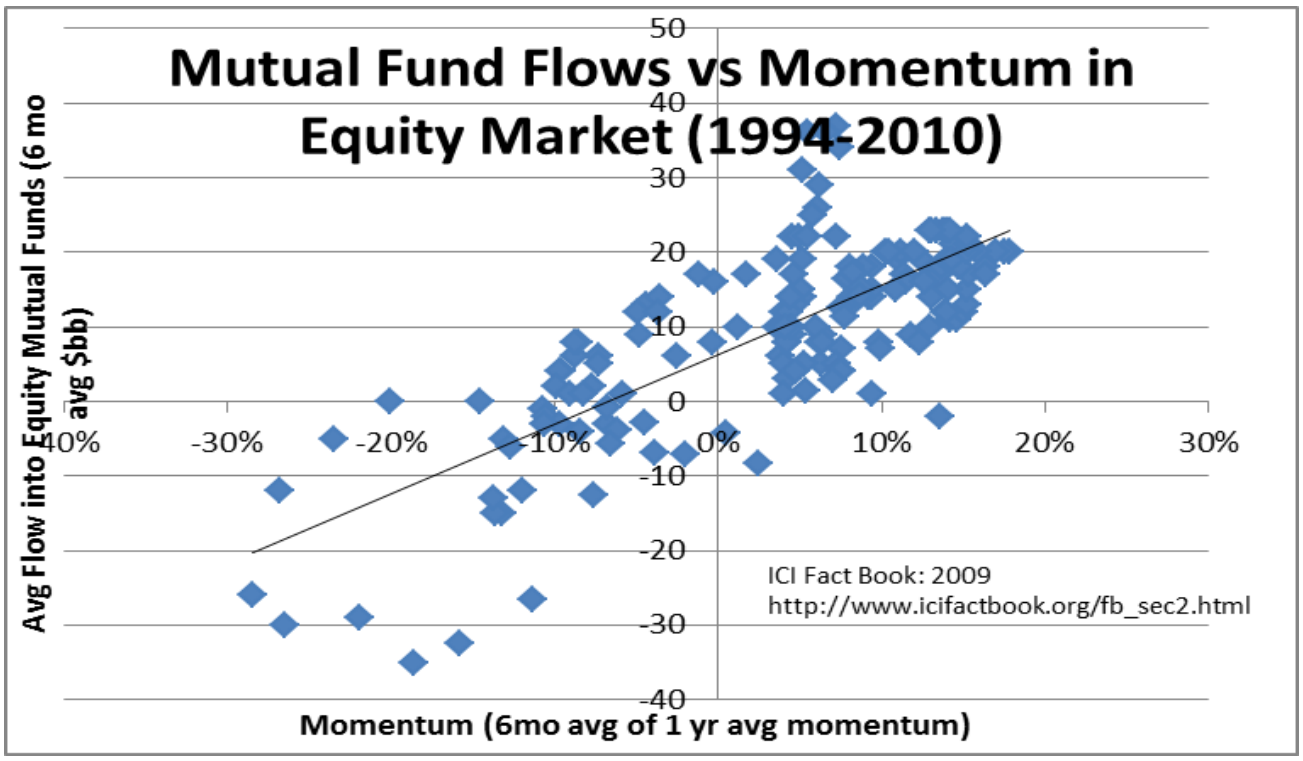
The chart below gives us some evidence for the type of behaviour that may lead to momentum effects in the market.[53] It indicates that net new flow into equity mutual funds is related to stock market

[50] George Soros' reflexivity, Irving Fisher, Hyman Minsky, JM Keynes, and many others.
[51] See various AQR/Assness research for excellent discussions of this area.
[52] From The Big Investment Lie, Michael Edesess, 2007, page 108.
[53] From ICI Fact Book 2010: http://www.icifactbook.org/pdf/2010_factbook.pdf (page 26).

performance, in this case, measured as the trailing 12 month return. Over the period, the correlation between momentum and flow of funds is around 0.5.

FLOWS TO EQUITY FUNDS RELATED TO GLOBAL STOCK PRICE PERFORMANCE

1 The total return on equities is measured as the year-over-year change in the MSCI All Country World Total Return Stock index.
2 Net new cash flow to equity funds is plotted as a six-month moving average
Sources: Investment Company Institute and Morgan Stanley Capital International



A number of different models of investor behaviour have been developed to try to explain the tendency of markets to trend. One such model is explained in a book by De Grauwe and Grimaldi called The Exchange Rate in a Behaviorial Finance Framework (2006). The authors put forward a model based on

the propensity of the investor population to switch between value oriented strategies and chartist, trend following strategies, depending on which strategy has been more successful recently. The model produces rich price dynamics with trends, bubbles and crashes.

Other explanations of the persistence of momentum in asset prices are anchoring, under-reaction to news, herding, and Keynes' beauty contest analogy. It is generally felt that simple trend following investment strategies have generated excess returns of about 1/3 to ½ of the annual variability (i.e. a Sharpe Ratio of .33 to 0.5). Our study generally conforms to those estimates.[54] Given a Sharpe Ratio of 0.40[55], we would expect to lose money in our momentum strategy 3 out of 7 quarters, and 1 out of 3 years. It is hard to attract and maintain investor confidence in a strategy with these characteristics if the only way that investors are judging the strategy is by how well it has been doing lately. It may be that the difficulty in investors believing in trend following as a fundamental characteristic of markets is itself an important factor supporting its persistence.

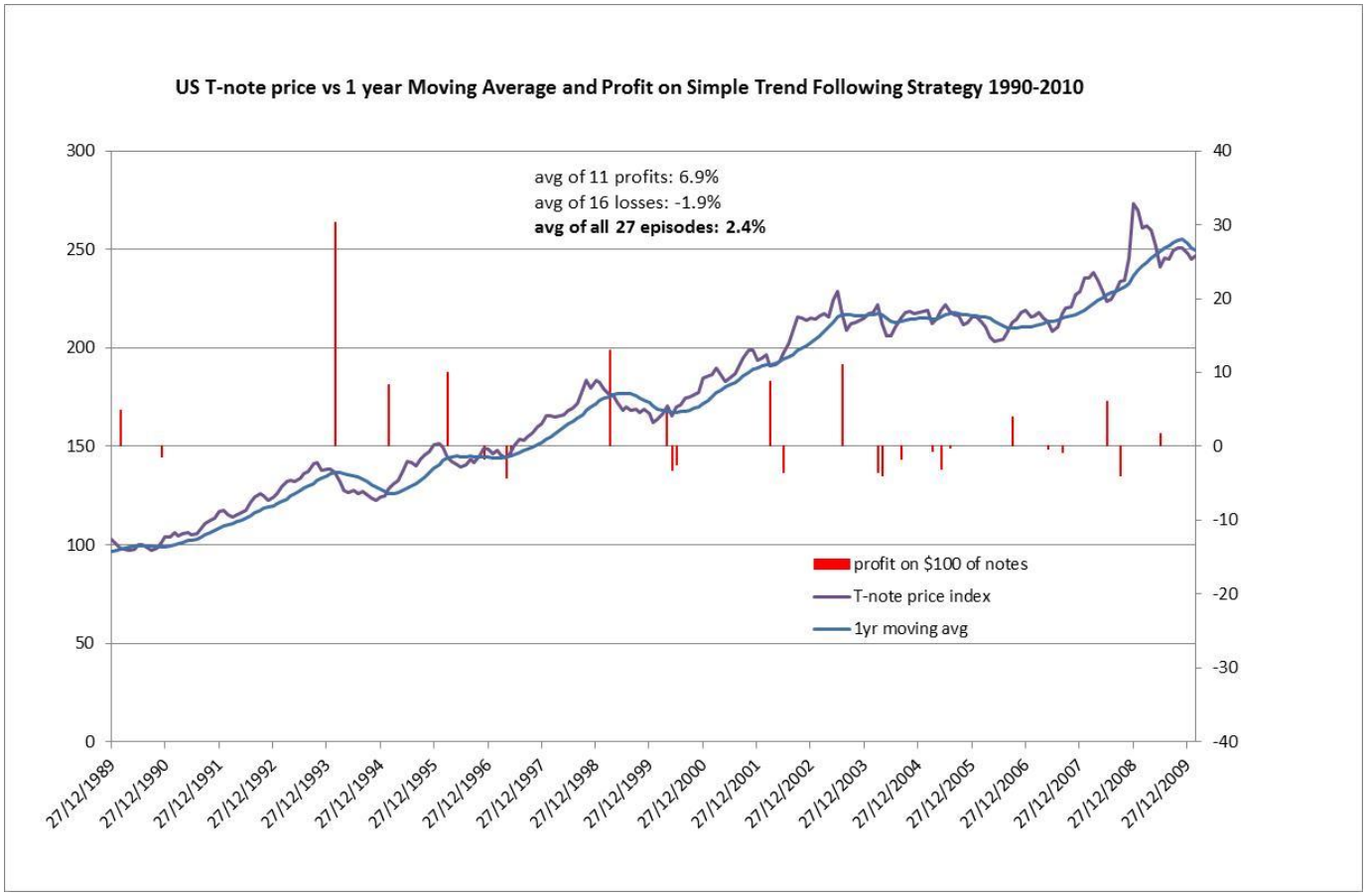
Interestingly, a distinctive common characteristic of some of the most successful macro fund managers is that they rely on stop losses. While investing with stop losses is not exactly the same thing as being a trend follower, it does tend to often align the positions of stop loss investors and trend followers as a practical matter, even if they arrived at their positions through very different thought processes. [56]

The use of stop losses is meant to produce a return pattern of frequent small losses when the stops are hit, hopefully compensated by less frequent, large gains. In the chart below we show the gains and losses from following a simple trend following strategy applied to US T-notes from 1989 to the end of 2009. The strategy is based on being long or short $100 of T-notes depending on whether momentum is positive or negative, which is much bigger than the momentum based scaling that we have implemented in the historical portfolio studies. Notice how the gains tend to be much larger than the losses. In fact there are more episodes of losses than gains, but overall the strategy is solidly profitable over this period.

[54] From 1926 to 2010 a momentum strategy based on US equities had a S.R. of .31 and for T-notes the S.R. was .45. From 1975 to 2010, US equity momentum had a S.R. of .14 and T-notes had a S.R. of .51.

[55] And ignoring transactions costs.

[56] An interesting study that we did was to compare the returns from a naïve strategy of macro trading using stop losses with a simple momentum strategy. We used an 8 asset portfolio (US equities, EM equities, US 10 year T-notes, 10 year investment grade credit spreads, commodities, real estate, and GBP/USD and JPY/USD exchange rates) from the start of 1975 to end of February 2010. The simple momentum strategy (go long when above (and short when below) the 1 year moving average, with a hurdle rate offset so that on average the strategy is not net long the asset) produced a return of 4.2% pa (real excess return over cash) with a Sharpe Ratio of 0.6. The naïve strategy was constructed by taking a series of randomly chosen long or short positions in the asset and exiting those positions when either the position lost money or when it made a profit of 1 annual standard deviation (so about 15% for US equities). The average return of the naïve strategy was roughly 3% with a Sharpe Ratio of 0.4. The correlation of monthly returns between the momentum strategy and the naïve stop loss strategy was around 0.3. On average the naïve strategy did run more of a net long position in the risky assets than the momentum strategy. We found the similarity between the two strategies very intriguing, and we feel there is much more to study in terms of identifying momentum strategies that naturally are embedded in forms of investment strategies.



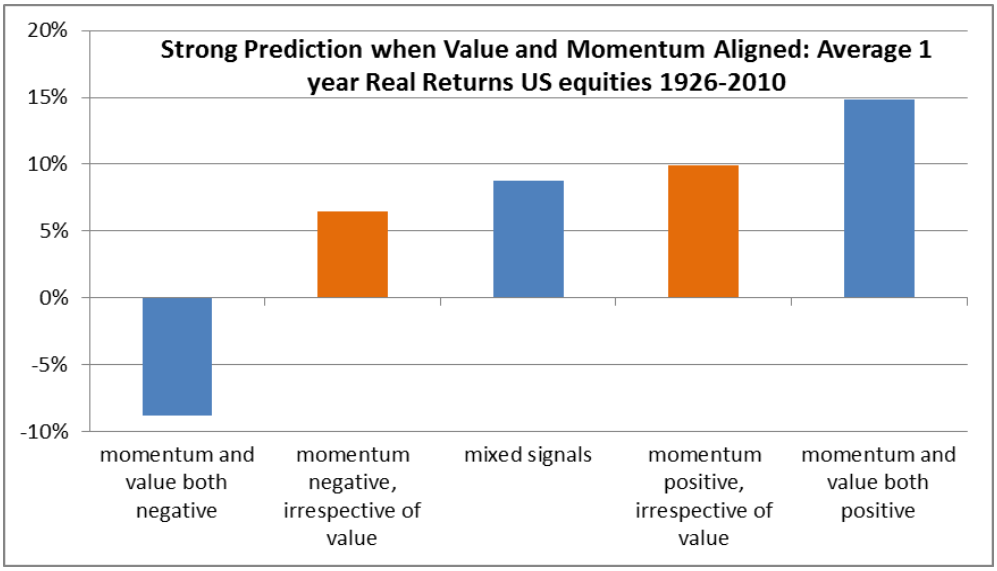
This positive assymetry in returns to a trend following investment style is a very attractive feature. In effect, losses are limited and gains are unlimited. It is intriguing to contrast this with relative value, or "arbitrage" investing, where the assymetry is usually in the opposite direction. When an asset becomes cheap, we buy it with the idea of selling it when it gets fair, which represents a fixed and limited upside. On the other hand, there is usually quite a long way[57] that the asset can go down, leading to a negatively assymtric return pattern for scaling based on value.
[58]

[57] Other than zero, and that is only in the case where we are buying an asset that is cheap. If we are selling an asset that is expensive, that richness is virtually unlimited.
[58] In deep value equity investing, the positive asymmetry is normally preserved, in contrast to normally leveraged relative value investing described.

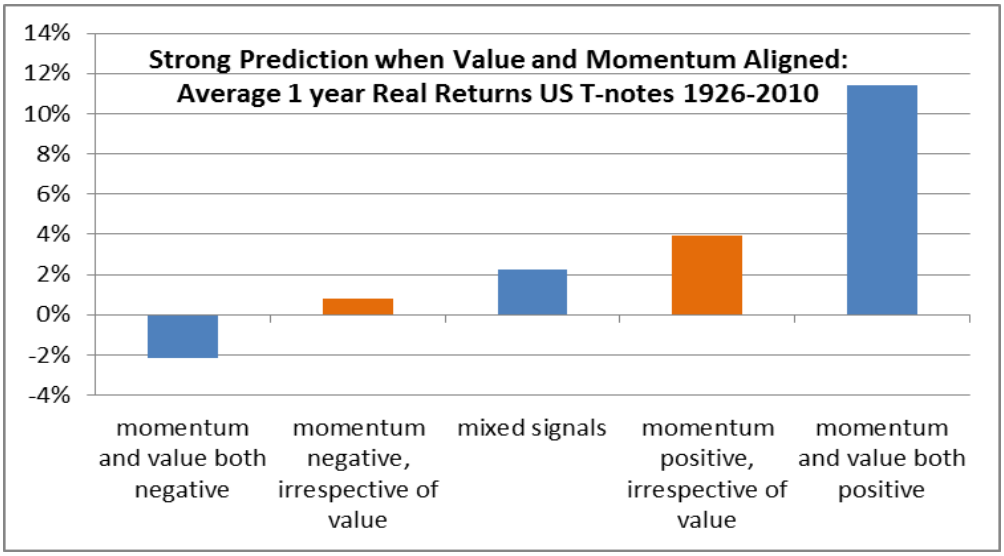
7) Relationship of value and momentum[59]

Value investing relies on reversion to the mean (if it went up in the past, it's more likely to go down next period), whereas momentum investing is based on the behavior that if it has gone up in the past, then it is more likely to go up again in the next period. How can these two views be reconciled? How can both things be going on at the same time? One possiblilty is that momentum and value are effective predictors to different investment horizons. We're not sure that's the case, as momentum certainly seems effective to a one year horizon[60] while value, in the few cases we've looked at, seems to be predictive of returns to a 1 month, 1 year and 5 year horizon.

Another possiblity is that much of the predictabilty of momentum and value are coincident. The two charts below give some support to that view, by showing that when value and momentum are giving the same signal, they tend to be more strongly predictive than either is on its own unconditionally. The blue bars cover all the possible combinations of the two signals, value and momentum, and the orange bars show the difference in return if we looked only at momentum, ignoring (or unconditional) on value. These results are quite encouraging, and perhaps could be explored further and lead to a better investment strategy than the one we report in this paper.

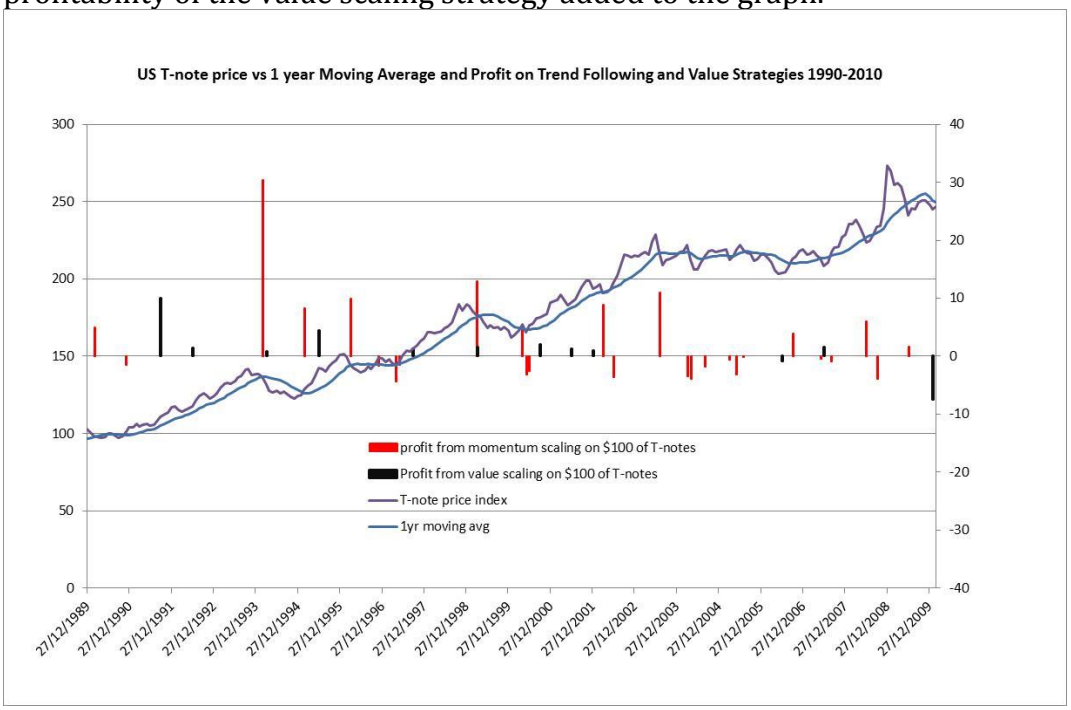


[59] Heavily influenced by AQR research and Value and Momentum Everywhere paper by Asness et al.
[60] Though not to a 5 year horizon.



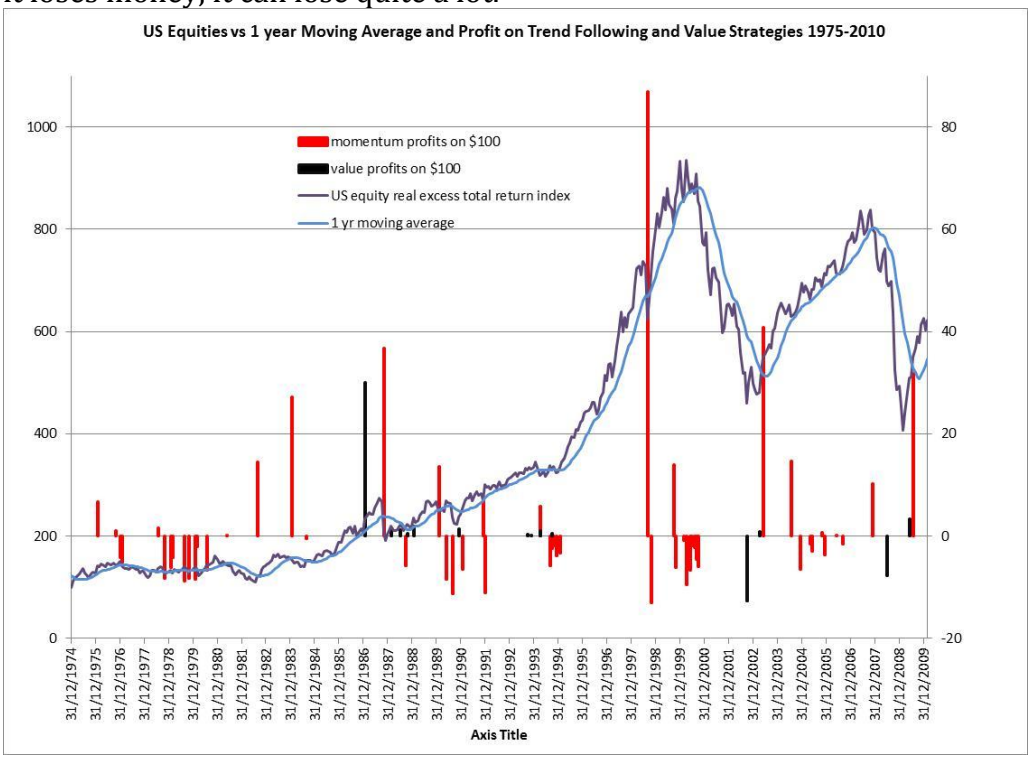
[61]

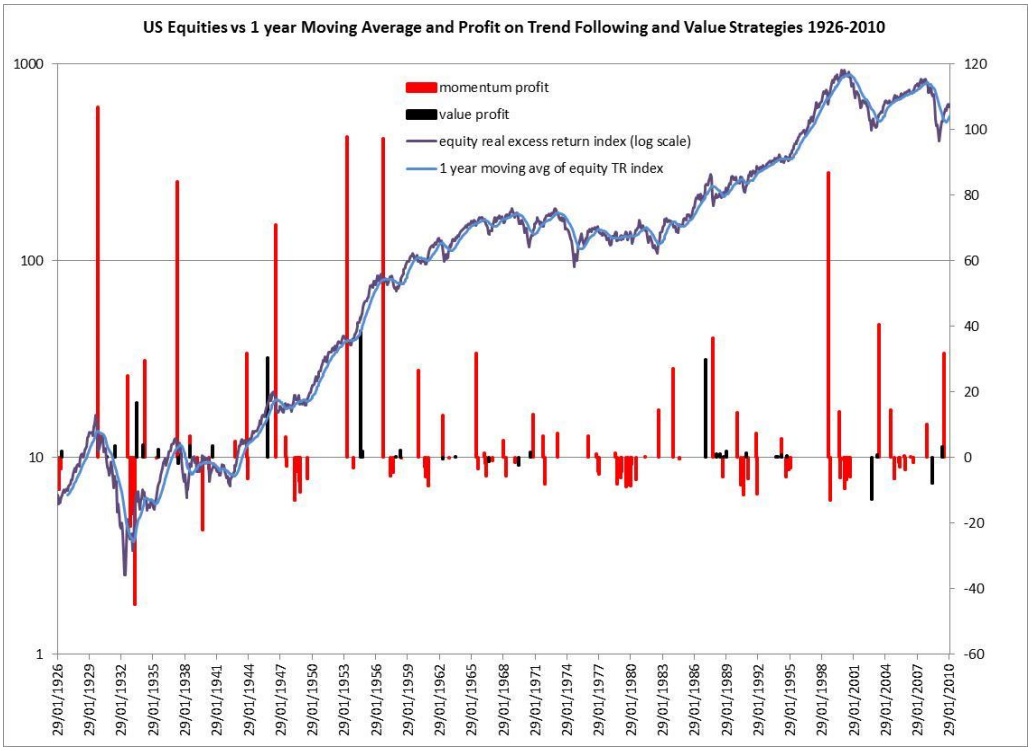
The chart below shows the profitability of the momentum and value strategies for US T-notes from 1990-2010. It is the same as the chart in the Momentum section above, but with the profitability of the value scaling strategy added to the graph.



[61] *Relating to this graph: when value was attractive and momentum was negative, the return was just as high as when value and momentum were both positive, which was somewhat surprising and disconcerting. Not so when value was negative. Worth studying this more- looking at other time periods and other assets, but still we believe the point from this chart is overall accurately stated.*

Below are two charts showing the same information for US equities for the two main periods we studied. Notice again the assymetrically positive returns for the momentum strategy and the more sporadic returns coming from value. Also notice the preponderance of positive episodes in the value strategy. As remarked earlier, it tends to have a higher hit ratio, but when it loses money, it can lose quite a lot.



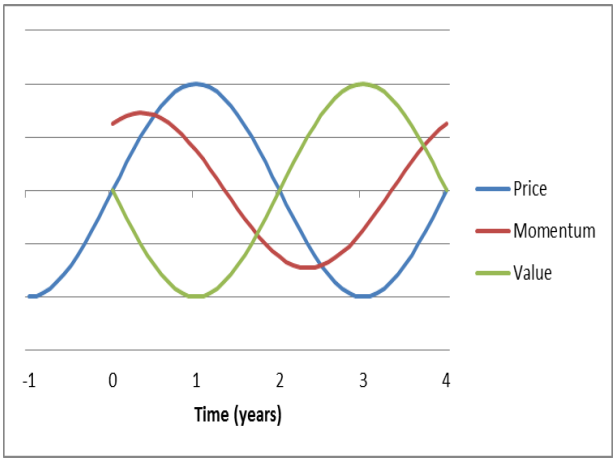


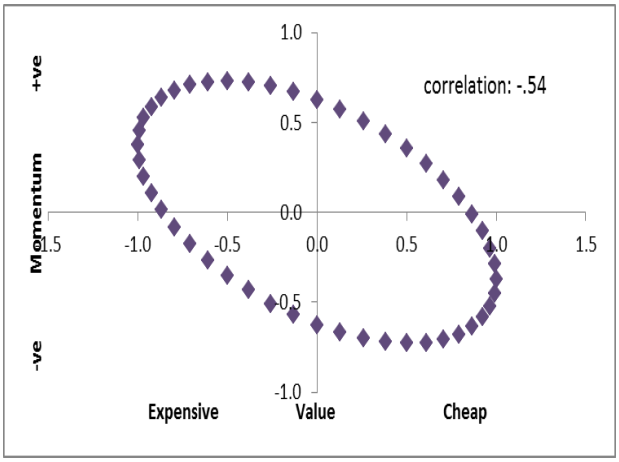
Notice in the chart above based on US equities from 1926 that the momentum strategy suffers a loss of over 40 points in April 1933 as equities exhibited extreme volatility. The years 1933 to 1937 were tumultuous, but overall through this period it was highly profitable as it captured the majority of the enormous 130% rally from mid 1935 to early 1937.

The correlation of returns on a momentum strategy with returns on a value strategy applied from 1926 to 2010 on US T-notes was -0.13, and for US equities over that period is was -0.36, which, while not very significantly different than zero, still represent a valuable characteristic within a portfolio and is consistent with the noticeably improved risk return characteristics when using value and momentum to increase returns above the buy and hold baseline[62].

Why is it that a value based strategy tends to be negatively correlated with a momentum strategy? The two charts below give a graphical representation of the relationship between value and momentum over a stylized valuation cycle of some hypothetical asset. The negative correlation in this case is a function of the parameter values chosen: a full valuation cycle of 4 years and a momentum strategy based on a one year moving average. It would be interesting to delve more deeply into the nature of the correlation in the two investment styles, which we intend to do in future research.

Negative Correlation Between Value and Momentum





8) Risk

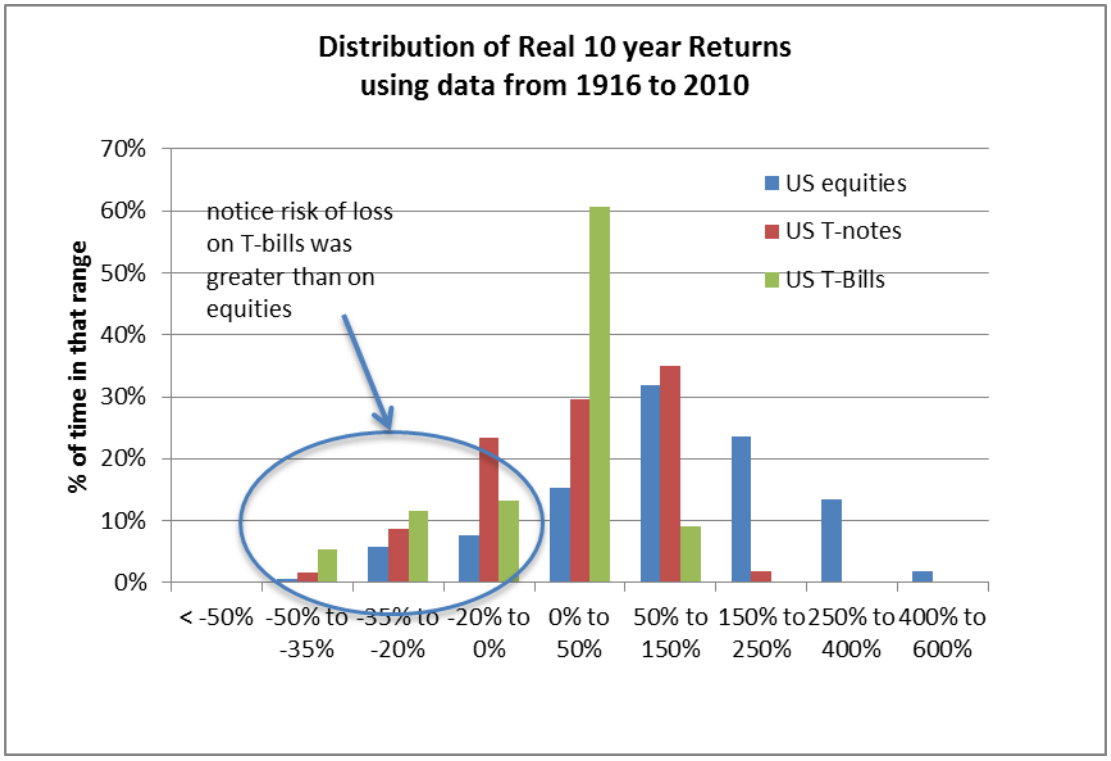
For most of us, when we think about the risk we are taking with our investment portfolio, we focus on the change in value of our portfolio over the next one to twelve months, using recent history to gauge how much we might lose. Given the relatively short horizon we're interested in, we normally don't worry about making adjustments for changes in the purchasing power of our portfolio. We tend to ignore the effect of inflation because it makes things more complex

[62] We should, and intend, to calculate the correlation between momentum and value strategy returns over more time periods and more assets.

for us to think about, and also, for the past 25 years in the developed world, inflation has been low and stable.

But what should we really be worried about when we think of risk? Shouldn't we be almost exclusively focused on the long term risk of loss in our portfolio, measured on an inflation adjusted basis? After all, isn't that the measure of risk which is most consistent with our investment objectives in the first place[63]?

As soon as we adopt an inflation adjusted, long horizon as the basis for our measure of risk, we find that our view of the riskiness of the main asset classes changes dramatically. Cash, or T-bills, which we may have considered the lowest risk asset, actually has quite a bit of risk to a long horizon measured on an inflation adjusted basis, and that is without even adding the risk of default or rescheduling of government obligations, which most people are no longer willing to completely rule out. Equities appear much less risky when viewed to a long horizon, as the extra expected return from owning equities gives a reasonable cushion against the greater variability of equity market returns in the short run. The graph below tries to illustrate the point using the past 95 years of data. Of course, the next 85 years is likely to be quite different than the past. Notice that the frequency of 10 year real losses on equities is less than that on T-notes which is less than on T-bills, a complete reversal of the normal risk ordering of those three asset classes.



[63] In general young investors should have a longer horizon than investors that are older. This life cycle adjustment to a balanced portfolio can easily be accommodated within the investment framework presented here. Also, it is interesting that an investor who is 80 years old still has a life expectancy of 10 more years, and that is not taking account of how bequests further lengthens an investor's horizon.

In practice, most investors find it near impossible to ignore the short run and focus solely on the long term, as it is the short run performance of our investments that confronts us most directly.[64] An investment approach which totally ignores short run portfolio risk will be counterproductive if the investor jettisons the strategy due to not being able to take the pain of large short term losses. Perhaps the biggest risk in investing is not being able to stay the course, either through lack of resolve or through being forced to liquidate due to the use of leverage in a portfolio.

This is why we are proposing an approach of a balanced portfolio which should have a short term riskiness that most investors can tolerate, while still generating healthy long term returns of the same order of magnitude as what would be expected from a portfolio invested 100% in equities.

Before leaving this topic, it is worth mentioning that there is another long term, inflation adjusted measure of risk that we should consider, namely, preserving the long term real annuity value equivalent of our wealth. Is it more important to us that our wealth stays constant in real terms, or is it more important to us that the real income that our portfolio generates stays constant[65]? Indeed this may be the most useful measure of risk for us to consider in structuring our long term portfolios, although it also suffers from the challenge that it would lead us to invest in such a way that many of us would perceive as too risky in the short run. In practice, investing with this view to the long term would require us to view inflation linked bonds (such as US TIPs) as the least risky of assets, probably followed by equities, and with cash and T-bills still being the riskiest from this perspective. What is better- a stable $1mm per year in real terms with the market value of our portfolio unknown or, on the other hand, a constant portfolio value of $15mm but with the real income stream it generates being variable?

9) So what can we expect from here?

The table below details the portfolio construction for the broadly diversified strategy, with value and momentum overlays, as of the end of February 2010. The portfolio is overweight equities, 64% rather than the 55% baseline, and overweight commodities.

[64] This excessive focus on short term risk may be one of the explanations of what is called "the risk premium puzzle", which poses the question of why equities have seemed to offer a return higher than warranted by reasonable levels of investor risk aversion.

[65] In England people used to refer to a person's wealth not in terms of "net worth", but rather in terms of a person's annual income, e.g. Darcy (from Jane Austen's Pride and Prejudice) was "worth 10,000 a year".

Index	Value Metric	value metric description	Base exposure	Value Signal	Value adjustment	Momentum Signal	Momentum adjustment	Total desired	Scaled down position
US equities	17.4	Shiller P/E	20.0%	5	0.0%	1	6.7%	26.7%	26.7%
European equities	15.2	Shiller P/E	9.0%	6	1.2%	-1	-3.0%	7.2%	7.2%
UK equities	11.4	Shiller P/E	5.0%	8	2.0%	1	1.7%	8.7%	8.7%
Japan equities	26.8	Shiller P/E	7.0%	2	-2.8%	1	2.3%	6.5%	6.5%
Pacific x-Japan equities	20.5	Shiller P/E	4.0%	3	-1.1%	1	1.3%	4.3%	4.3%
Canada equities	23.3	Shiller P/E	2.0%	3	-0.5%	1	0.7%	2.1%	2.1%
Emerging Market equities	20.4	Shiller P/E	8.0%	3	-2.1%	1	2.7%	8.5%	8.5%
Total in Equities			**55.0%**		**-3.3%**		**12.3%**	**64.0%**	**64.0%**
% in each equity market in Value sector:									
US	-2.9%	past 3 year return vs broad	10.0%	7	2.7%	1	3.3%	16.0%	16.0%
Europe	-3.5%	past 3 year return vs broad	4.5%	8	1.8%	-1	-1.5%	4.8%	4.8%
UK	-3.0%	past 3 year return vs broad	2.5%	7	0.7%	1	0.8%	4.0%	4.0%
Japan	2.9%	past 3 year return vs broad	3.5%	4	-0.5%	1	1.2%	4.2%	4.2%
Pacific x-Japan	-1.8%	past 3 year return vs broad	2.0%	7	0.5%	1	0.7%	3.2%	3.2%
Canada	0.5%	past 3 year return vs broad	1.0%	5	0.0%	1	0.3%	1.3%	1.3%
Total in Value			**23.5%**		**5.2%**		**4.8%**	**33.5%**	**33.5%**
Real estate	6.3%	dividend yield	10.0%	3	-2.7%	1	3.3%	10.7%	10.7%
Commodities		combined: price / 10yr avg px and backwardation	10.0%	6	1.3%	-1	-3.3%	8.0%	8.0%
Interest rate risk of which:	1.65%	expected real rate	15.0%	2	-6.0%	-1	-5.0%	4.0%	
10 year T-note			10.0%						0.0%
Investment Grade Corporate bonds	1.70%	credit spread	5.0%	5	0.0%	1	1.7%	6.7%	4.0%
Cash			10.0%						13.3%
Total			**100%**		**-3.6%**			**86.7%**	**100%**

It is underweight interest rate risk, with only 4% of the 15% baseline exposure, all allocated to corporate bonds[66], and all together, cash is 13.3%% of the portfolio, as opposed to its 10% baseline weight. Value equities are above their baseline weightings too, and represent more than half the total equity market risk.

This allocation is telling us that we should expect above average returns from risky assets in the period ahead (ahead from February 2010 that is). Estimating the long term return from this strategy is a fairly complicated task, as future returns will be generated from the buy and hold as well as dynamic characteristics of the investment strategy. Using the past as a guide, it seems that we should be able to increment a pure buy and hold return by 2-3% pa, before transactions costs. What we can do, with considerably less difficulty, is estimate the long term real return from the baseline buy and hold portfolio at any point in time. For most components of the portfolio, equities, government and corporate bonds and real estate (less so with commodities and value equities) we can have some confidence that in the very long term, our valuation metric is related to the long term return to be expected. It is interesting to note that

[66] The table helps us to illustrate a slight complexity in how we construct the portfolio with regard to allocation to value equities and to corporate credit. Notice that the allocation to value equities is termed "% of each equity market in Value sector". We first decide how much equity risk we want to have and then our value equities allocation is an overlay where we take some, or all, but not more than, the total equity market risk in each market and express that through owning value equities. It is a similar process with interest rate risk; first we decide how much interest rate risk we want and then we decided how much of that should be in the form of corporate bonds, the rest being in the Treasury note. What this means is that sometimes, when equities or bonds in general are not attractive because of high P/E's or low expected real returns, respectively, we cannot have the allocation we might want to have to Value equities or to Corporate bonds. We could relax that constraint, by allowing an allocation in value equities and then a short position in the broad market, but for the analysis in this paper, we have kept that constraint.

it is generally considered easier to estimate long term returns than short term returns[67]; sadly, most investors are heavily influenced by short term risk and return.

For instance, it is intuitively appealing, and the evidence of the past 100 years supports the view, that corporate earnings and dividends, cyclically adjusted (as in using the past 10 years average), keep pace with inflation but lag real economic growth. Over the past 100 years that shortfall in dividend growth relative to real GDP growth has been in the range of 2 ½ -3 ½% per annum[68]. That this "slippage", as the lagging of public market equity earnings and dividends is sometimes called, exists seems to make sense given that there are a number of new start-up companies, big and small, that eat into some of the profits that publicly held companies would have been able to make. There are other sources of slippage too, such as dilutive capital raises from big companies that go to a select few non-public investors, particularly in times of crisis. However, the large magnitude of the slippage is quite surprising and fairly sobering in terms of the prospects for future real equity returns.

Readers will have noticed that there is one asset in our portfolio whose long term real return we do not try to estimate- cash. We treat cash, or, more precisely, T-bills, as the residual allocation in our strategy. Perhaps trying to estimate the long term return from holding cash should be part of our decision process.

Once we have made a bottoms-up estimate of the long term real return expected of our buy and hold baseline portfolio, perhaps the simplest approach is to add a bit of return on for the value and momentum overlays. Perhaps when the baseline portfolio has a low expected return, those overlays would be expected to add more; this is something we should, but have not, researched.

What can we expect for risk going forward from here? Probably the Three Country study from 1926 to 2010 gives us the best information to go on. The interconnectivity of the global economy is greater than it has ever been, but at the same time there are more asset classes and groups available to the investor for diversification. Taken together, we believe the benefits of diversification will be less than they were historically and indeed risks such as we have seen in the 20 years leading up to WWII should not be ruled out, or even heavily discounted. Certainly the recent experience of the 2008/09 financial crisis gives us a taste of how bad things can get and how quickly, and we must consider that that episode could have been significantly worse had it not been for large scale government intervention to stabilize asset prices. In thinking about risk going forward, however, it is important for us to really know what form of risk is most important to us. If it is risk to long term real returns on our portfolios, rather than the

[67] See earlier chart of 30 year US equity returns as a function of P/E in the "Simple Measures of Value" section above.

[68] In Triumph of the Optimists (Dimson, Marsh, Staunton) they calculate that real dividend growth from 1900-2000 in the US and the UK was 0.6% and 0.4% pa, and the average for all 16 countries they investigated was -0.5% over the 100 year period. This was a lag of 2.6% and 2.0% in the US and UK versus real GDP growth, and a lag of between 3% and 4% for all the countries on average. From 1950-2000, real dividend growth was 1.4% over all 16 countries and 0.6% and 2.1% in the US and UK. This was probably a lag of around 2% versus real GDP, although DMS do not provide the real GDP comparison. Our data is similar in conclusion to the DMS data, indicating that in the US over the same period dividends had a 1.2% real annual growth, which lagged real GDP by 2.4% pa. These slippages over the past 100 years are very sobering in terms of the prospects for future real equity returns.

risk to the nominal present values, perhaps we will find that our portfolios have less risk in them than we otherwise believed to be the case.

10) Relevant investment funds, ETFs and other vehicles

While there are a variety of ways of implementing the strategies outlined in this paper, we have set out below a list (as of February 2011) of a set of index funds and exchange traded funds (ETFs) which can be used to that end. Annual fees and transactions costs in general on these investments is very low, and has been falling over the recent past as more assets have gravitated towards these vehicles, resulting in economies of scale in terms of management fees. Also, the performance of these funds versus their benchmark indices have been improving as the managers become more savvy in following strategies to reduce or eliminate drag, such as participating in the price pressure impact of index inclusion and exclusion flows and lending out of securities for a fee.

Ticker	Description	annual fees	ETF or Fund	Family	Outstanding Market Value $BB	# of securities in portfolio	average maturity (years)
VTI	US equities: total market	0.07%	ETF	Vanguard	19	3389	
VEA	European equities	0.16%	ETF	Vanguard	6	481	
VPL	Pacific equities	0.16%	ETF	Vanguard	1.6	493	
EPP	Equities: Asia x-Japan	0.50%	ETF	Blackrock/ ishares	4	152	
EWC	Equities: Canada	0.53%	ETF	Blackrock/ ishares	5	101	
VWO	Emerging equities	0.27%	ETF	Vanguard	43	892	
VTV	US equities: value- MSCI Prime Market	0.14%	ETF	Vanguard	5	417	
VTRIX	International equities Value	0.45%	Fund	Vanguard	8	250	
VNQ	Real estate: US	0.13%	ETF	Vanguard	8	104	
VNQI	Real estate: non US	0.35%	ETF	Vanguard	0.1	424	
GSG	Commodities: GSCI index	0.75%	ETF	Blackrock/ ishares	2		
DBC	Commodities: DBIQ index	0.85%	ETF	Deutche Bank	5		
DJP	Commodities: Dow Jones-UBS index	0.75%	ETF	iPath	3		
VFIUX	US Treasury intermediate term	0.12%	Fund	Vanguard	6	39	5.8
VAIPX	US inflation protected bonds	0.12%	Fund	Vanguard	32	33	8.7
VWIUX	Municipal bonds: intermediate maturity	0.12%	Fund	Vanguard	29	2875	6.7
VFIDX	Corp. bonds: Intermediate Investment Grade	0.11%	Fund	Vanguard	15	1175	6.7
VWEAX	High yield US bonds	0.15%	Fund	Vanguard	13	321	6.4
	other possible investments for further diversification						
VB	US equities: small cap	0.14%	ETF	Vanguard	5	1737	
VSS	Non-US world small cap	0.40%	ETF	Vanguard	0.9	2700	
IWN	US small cap value (Russel 2000 value)	0.41%	ETF	Blackrock/ ishares	5	1294	
TIP	US inflation protected bonds	0.20%	ETF	Blackrock/ ishares	19	31	8.7
GLD	Gold	0.40%	ETF	State Street	53		
CEW	Emerging market currencies	0.55%	ETF	WisdomTree	0.4		
ELD	Bonds: emerging market local currency debt	0.55%	ETF	WisdomTree	0.6		
EMB	Bonds: emerging market	0.60%	ETF	Blackrock/ ishares	2	75	11.8

11) Transactions Costs

The historical returns presented above have all been index returns, without any subtraction for transactions costs. In practice, an investor will bear transactions costs in the form of fees, commissions and bid-offer spreads. As can be seen from the table above, the bulk of a portfolio would consist of funds and ETF's with annual fees less than 0.2%, although there would be some component of the portfolio with annual fees as high as 0.8%. It is felt that transactions costs in total would subtract less than 0.3% per annum from the returns that would be generated in the absence of all transactions costs. To be added: futures, closed end funds, high cash rates, etc.

12) Return Enhancements

While fees and other transactions costs put a drag on returns, it should be possible for a long term investor to periodically add a bit of return to her portfolio by a judicious sprinkling of investments that trade at a discount to their underlying value. One possible source of this extra return might be found in closed end funds available at a steep discount to underlying net asset value. Other possibilities might include the possible use of futures when they are rich or cheap to the underlying asset or cheap cash equivalent investments, such as over-collateralized secured overnight lending (ie. repo's).

13) Alternative assets

Although a number of very successful and high profile investors (the Yale and Harvard University Endowment Funds) have achieved high quality investment performance by investing heavily in alternative assets, we have decided to exclude them from this study for the following reasons:

1) Investing in alternatives requires a commitment of quite a lot of resources to identify, or get access to, the successful managers. Paying someone else to perform those functions, such as a fund of funds, has not produced very attractive returns.
2) Historical data on alternative assets is not readily available prior to 1990 and the quality of the data is notorious for overestimated returns due to various biases in the construction of the databases, such as backfill and survivorship bias.
3) Historical returns for hedge funds, private equity and venture capital as overall asset classes, do not look very attractive on a historical basis, although there does seem to be some persistence in returns for funds that have been successful in the past[69]. Getting direct access to those top performing funds is difficult and/or costly, and not practical for most investors.
4) It may be that a good part of the special returns generated by alternative investments are captured by the approach outlined in this paper. For instance, exposure to value stocks and momentum are two relatively common sources of return in the alternative space.
5) Creating a diversified portfolio of alternative assets, without relying on fund of funds, requires a large amount of capital, as most funds have fairly significant minimum investment requirements.

[69] Note to follow up on: Sandhill Econometrics has some data, and Dow Jones may be publishing it.

For all these reasons, we felt that it was not practical or desirable for ordinary investors to count on investing in alternatives as part of their long term investment strategy. We have included return data from the HFR database from 1990-2010 and 2000-2010 in our summary of the various asset class returns, even though the data were not used in any of the strategies.

14) Exchange rate risk, and return

All of the historical results reported is based on a passive approach to taking foreign exchange risk. All returns are reported in US dollars and the nominal amount of foreign exchange risk taken is equal to how much is invested in each non-dollar denominated equity market.

There is a body of literature which shows that historically there has been a degree of predictability in future foreign exchange rates based on, among other factors, carry, purchasing power parity and momentum. We have taken a brief look at just a few currencies (British Pound, Japanese Yen, Euro[70] and Canadian dollar). We did not investigate carry as a factor, as we felt that even if we believed it to have predictive power, that it also has the negative characteristic that carry based strategies suffer heavy losses at the same time as equity markets.[71] Also, we wanted to keep our analysis focused on the value and momentum metrics that we were using in the rest of our asset allocation framework.

The results that we found were broadly consistent with other research and with our expectations. Value, when applied to foreign exchange rates by comparing today's rate with the ten year inflation adjusted rate or with an OECD measure of PPP (purchasing power parity), was overall a positive strategy since 1975, but not by a lot. It gave positive results in the case of the Pound, the Yen and the Canadian dollar, but not so in the case of the Euro. The S.R. in all cases was less than 0.2.

On the other hand, momentum produced solid returns in all four currencies, with S.R.'s of around 0.4 for all the currencies, except the Canadian dollar at around 0.2. Also, these results held in the case of the Pound and the Yen back to 1926, although of course the data and the differing exchange rate regimes makes that record of less relevance.

As in other cases, the correlation between momentum and value returns was negative, averaging around -0.5, which is an attractive feature of combining value and momentum, as mentioned previously. When added to the broadly diversified portfolio following the value and momentum strategy (1975-2010), we found that the currency overlay strategy added return and left risk virtually unchanged- a good preliminary result in terms of the historical record.

We look forward to doing more work on the issue of foreign exchange risk in the asset allocation framework.

[70] Spliced with the Deutsche Mark (check this) before its introduction.
[71] This insight comes from Antti Ilmanen, see Expected Return.

15) Taxes

All of the analysis presented in this paper was done on the basis of an investor who paid no taxes. While this is a good description for some investors (or for some of an investor's assets, such as those within a tax exempt pension plan), it is not representative of the bulk of affluent individual investors in the US or UK. Taking account of the effect of taxes is a complex task, and beyond the scope of this paper. However, a few general points on the effect of taxes on investment strategy are as follows:

1) Fixed rate bonds and cash are more heavily taxed than equities, and so on an after tax basis, the returns from equities are even more attractive than returns from cash or bonds on a real, after tax basis. In the UK, inflation indexed bonds get a more favorable tax treatment than nominal bonds, and in the US, municipal bonds can be an attractive replacement for a combination of government and corporate bonds
2) In general, longer term holding periods reduce effective tax rates, and so portfolio turnover incurs additional costs beyond the normal direct transactions costs. The dynamic strategies presented in this paper based on value and momentum would need to be less dynamic, or perhaps separated for tax purposes from the core buy and hold portfolio (perhaps through the use of imperfect futures hedges) to produce benefits after taking into account taxes.
3) Tax rates and regimes change with regularity and investors will need to monitor and react to those changes.

16) Further research

While there is a near limitless amount of further research we would like to do on this topic, the following are some of the areas we'd like to explore in the near future:

- Looking at a strategy that integrates value and momentum more fully, recognizing more directly that when value and momentum point in the same direction, the signal is more predictive than either on its own
- Evaluating other rebalancing frequencies, such as quarterly or annually, and also other rebalancing schemes, such as only making changes to the portfolio when they are beyond a certain threshold.
- Introducing other assets into the analysis such as
    - Inflation indexed bonds
    - High yield bonds
    - Emerging market bonds
    - Municipal bonds (in the US)
    - Mortgage backed bonds
    - Non US bond markets
    - Small cap equities
    - Foreign real estate
    - Utility stocks
    - Natural resource and commodity focused companies
    - Options
- Giving more direct consideration to the effect of foreign exchange risk on the portfolio and in particular exploring whether applying value (such as purchasing power parity)

and momentum metrics to foreign exchange exposure can increase portfolio returns and reduce risk

- Evaluating other simple momentum strategies, such as running three windows (1 year, 6 months and 3 months) rather than the single one year moving average, or alternatively, rebalancing the value oriented strategy more with a time lag

17) Conclusions

Using crude measures of value and momentum to directly allocate investments across asset classes has historically produced superior returns and lower risk than the other simple investment strategies we considered. This research is distinctive in that we have explored these effects at the level of asset classes, rather than the more prevalent existing body of research which has focused on value and momentum at the level of individual securities within asset classes, and usually would require investors to use leverage and short positions in a high turnover strategy.

The crude but intuitive metrics we use for value allocations allow investors to avoid allocating to assets that are in extreme cases of overvaluation or "bubbles" while giving investors confidence in holding assets that are undervalued when panic reigns in the markets.

Similarly, our simple metric for momentum allows investors to capture the long observed benefits of momentum that have appeared across assets and time periods. This momentum overlay imparts a kind of stop-loss discipline to the investment process. It is fascinating to note that many successful traders and risk managers rank a stop-loss approach (taking losses quickly and letting profits run) as the single most important ingredient in long term investment success.[72]

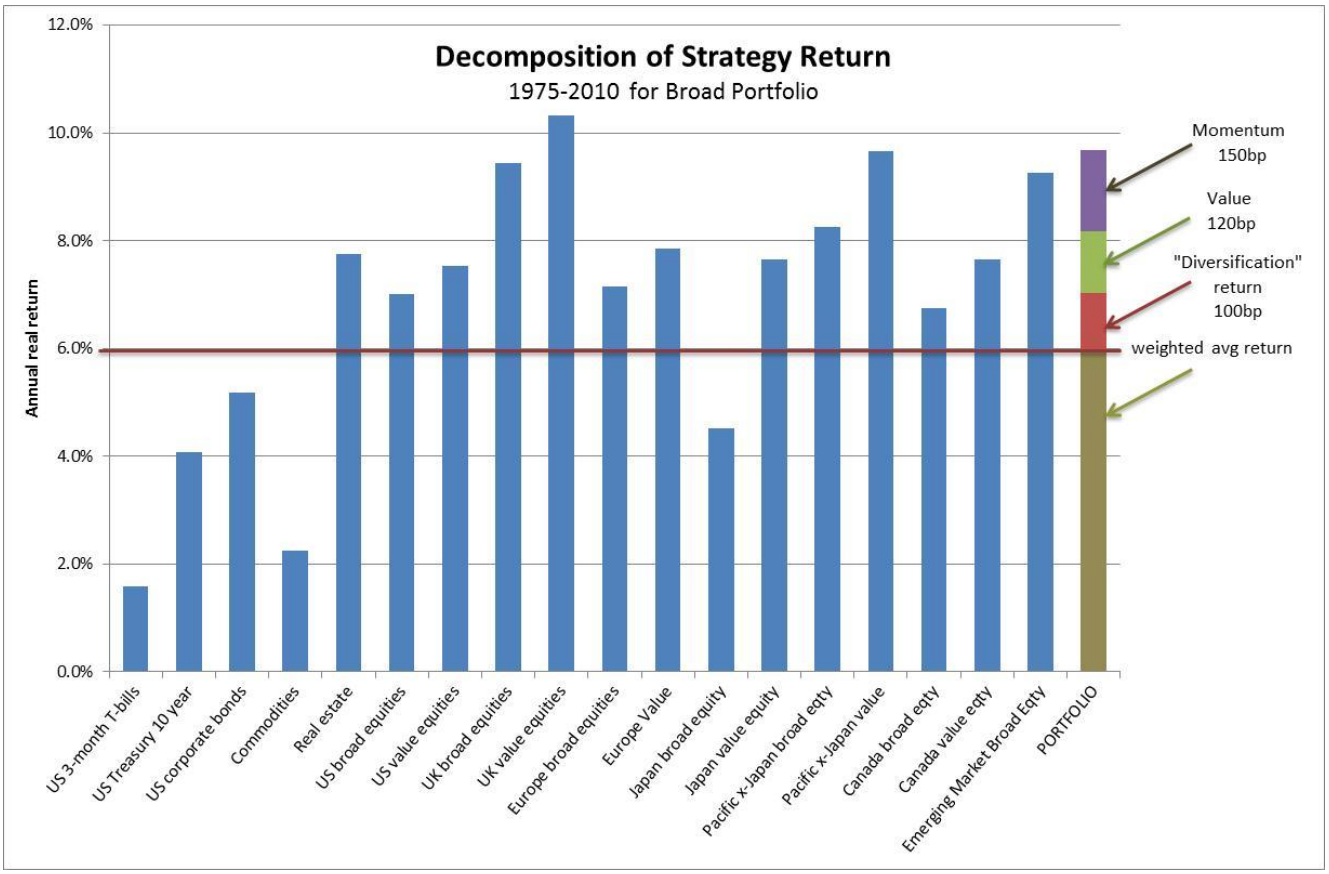
We find that value and momentum provide the best returns when used in combination. This is due to the negative correlation and general complementary nature of value and momentum.[73] However we still find that either value or momentum used alone provides better investment results over a long horizon than using a buy and hold strategy.

We find that value and momentum decrease the risk in a given portfolio. Across all of our studies we find that a strategy incorporating a dynamic allocation to assets based on value always produces a better reward to risk tradeoff than a buy and hold strategy. Employing momentum and value was historically much more effective than using value alone in all of our studies. Again we find this to be true across time, markets and assets. The charts below give a summary of our findings for the broadly diversified strategy run from 1976 to the present.

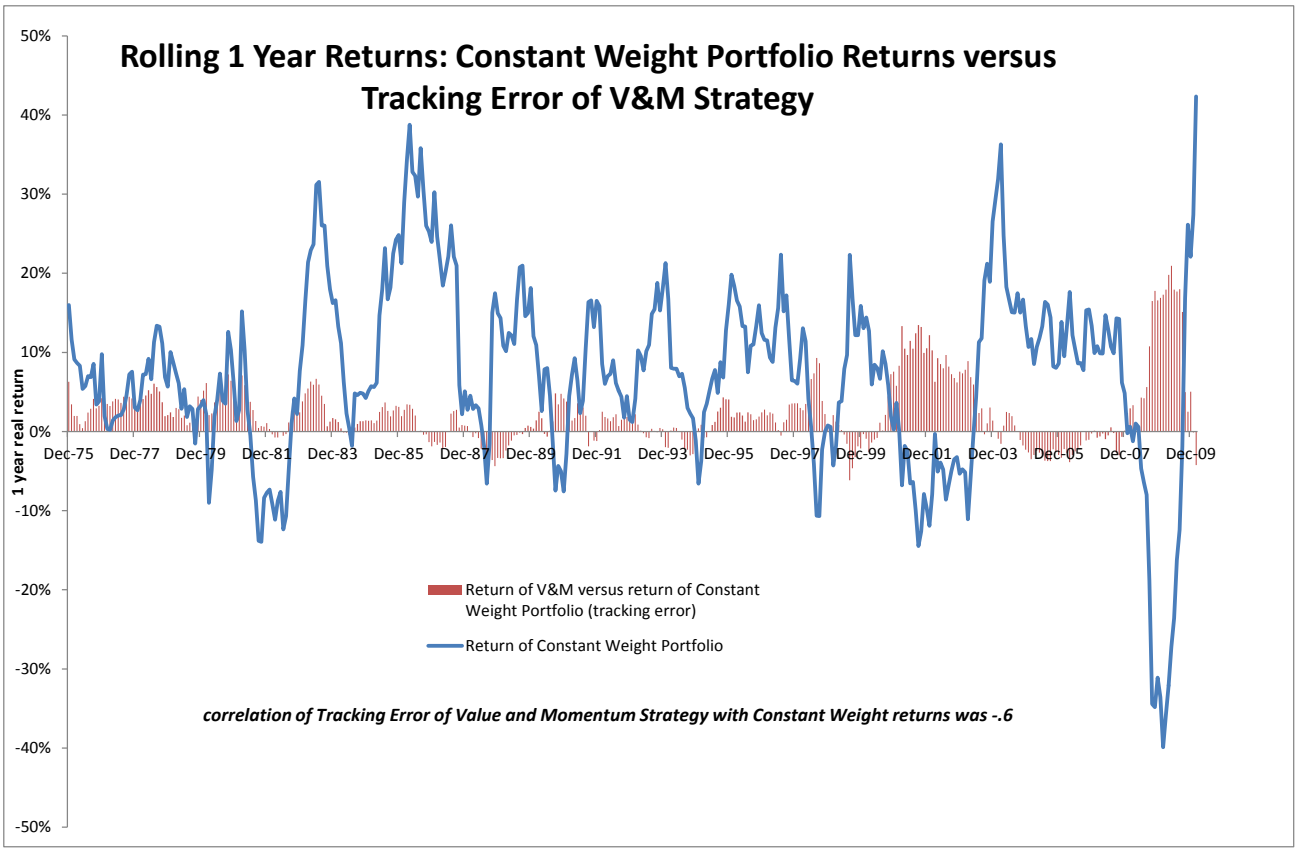
As we have discussed more fully in the body of this paper, we believe that value and momentum effects will persist in the future and allow investors who gain low cost and

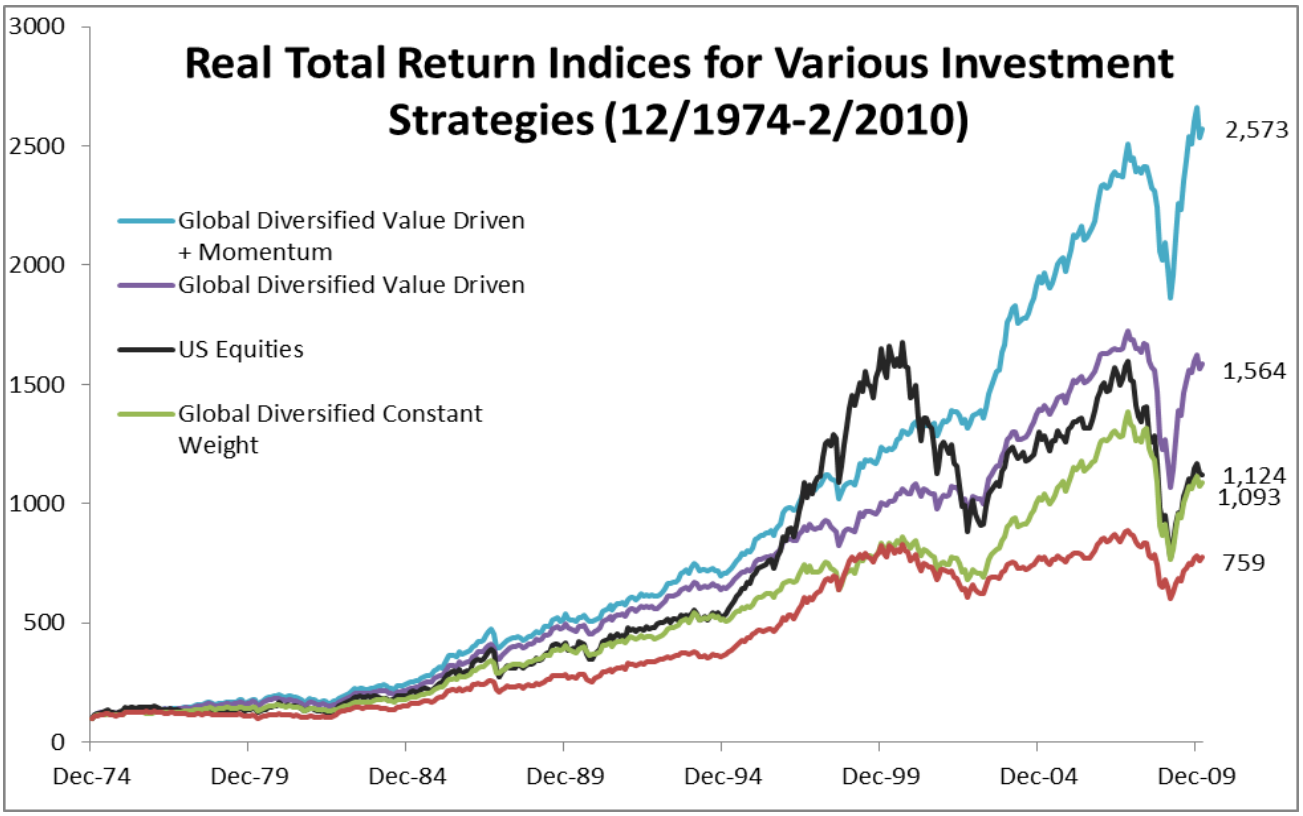
[72] See Jack Schwager's classic interviews with prominent investment managers in Market Wizards (1989) and New Market Wizards (1992).
[73] Again, see Asness, Moskowitz, Pederson 2008.

disciplined access to these sources of return to enjoy attractive long term returns relative to more passive, indexed alternatives.



	1975-2010	1990-2010	2000-2010	worst 1 year	worst 5 years (total return)	worst 10 years (total return)	Standard Deviation (annualized)	Return vs Constant Weight	Tracking Error vs Constant Wght*
Constant Weight	**7.0%**	5.0%	2.7%	-40%	-18%	8%	10%		
Value Only	**8.2%**	5.9%	4.6%	-35%	-18%	21%	10%	1.1%	2.7%
Value and Momentum	**9.7%**	8.1%	7.5%	-23%	2%	73%	9%	2.6%	3.5%
Inflation	4.1%	2.7%	2.5%						


Rolling 1 Year Returns: Constant Weight Portfolio Returns versus Tracking Error of V&M Strategy
1 year real return
50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
Dec-75
Dec-77
Dec-79
Dec-81
Dec-83
Dec-85
Dec-87
Dec-89
Dec-91
Dec-93
Dec-95
Dec-97
Dec-99
Dec-01
Dec-03
Dec-05
Dec-07
Dec-09
Return of V&M versus return of Constant Weight Portfolio (tracking error)
Return of Constant Weight Portfolio
correlation of Tracking Error of Value and Momentum Strategy with Constant Weight returns was -.6


Real Total Return Indices for Various Investment Strategies (12/1974-2/2010)
3000
2500
2000
1500
1000
500
0
Global Diversified Value Driven + Momentum
Global Diversified Value Driven
US Equities
Global Diversified Constant Weight
2,573
1,564
1,124
1,093
759
Dec-74
Dec-79
Dec-84
Dec-89
Dec-94
Dec-99
Dec-04
Dec-09

Appendices

1) *Defining the baseline portfolio*

An early and powerful insight from modern financial theory is the idea that all investors' portfolios should be made up of some amount[74] of the risk free asset with the rest invested in a market capitalization weighted blend of all available risky assets, known as "the market portfolio". This is a very elegant theory, and several Nobel prizes were awarded to its creators. Over time many of the assumptions it was built upon have come to be viewed as unrealistic. Even if we accept most of the assumptions[75], we run into trouble as soon as we try to implement it. The biggest practical problem of building a market portfolio is the difficulty in dealing with the large amount of risky assets not available for purchase by the public.[76] These non-publicly available assets include privately held businesses, privately held real estate (residential, commercial and agricultural), and government owned assets. If investors ignore those assets, in effect they do not own the market portfolio, and on the other hand, if enough investors really did take them into account and try to make their weightings in publicly available assets mirror the totality of the market for risky assets, it just wouldn't be possible because there wouldn't be enough of some of the publicly traded assets to go around. The table below gives an idea of the challenges involved in deciding on the weights to use in establishing a market portfolio based on market capitalization weights, even assuming that we can accurately quantify the value of each of these categories. Interestingly, one of the most commonly suggested asset allocation baselines, 60% in equities and 40% in bonds, is not within any of the ranges of values that we would come to using today's market capitalizations of the various asset classes.

[74] Can be a negative amount, representing leverage.
[75] Or refine the theory into more general forms, such as the multi-risk factor Arbitrage Pricing Theory (APT).
[76] There are myriad other practical problems in implementing a true market weighted portfolio of risky assets, including such issues as whether an asset should be included at all (e.g. are government bonds just a debit and credit that net off to zero?), determining the size of various non-publicly quoted assets, foreign exchange risk making assets have different risks depending on the investor's base currency, the significant challenges posed in integrating a concept of human capital into the analysis etc.

			as of end of 2009
Estimate of Market Value	Estimate of weight based on publicly held values	Estimate of weight based on non-publicly held values	Asset (global)
			Publicly held assets
31	27%	14%	Equities
0.3	0%	24%	Real estate through REITs and other companies
33	29%	10%	Government bonds
49	43%	14%	Corporate and other non-govt bonds
		24%	Commodities
		15%	Other
113	100%	100%	**Total Publicly available**
			Non Publicly held assets
17			Equities
58			Residential real estate
21			Commercial real estate
4			Agricultural land
75			Oil
6			Gold
50			other assets/ commodities / etc
230			**Total Non-Publicly available**
344			**Grand total**
all figures in $Trillions			
global GDP end 2009		58	

As mentioned earlier, it is not even clear that investors should want to be invested in the market portfolio, passively increasing and decreasing their exposure to assets as their weights in the market portfolio go up or down. There are two ways that an asset's weight can change: 1) the price of the asset goes up or down relative to the other assets, or 2) more of the asset becomes available to investors. Another problem with passive market capitalization indexing is sometimes known as the "bum" effect and it can be particularly pernicious in indexing to bonds. As governments or businesses issue more bonds, and become more indebted and possibly less likely to be able to repay their debts, the weight of those bonds in the index increases. In such a way, investors following passive indexation find that they have to invest more in bonds, and less in everything else, as governments run higher deficits, and in particular, they have to invest more in the bonds of those very governments who are issuing the most. This hardly seems like good investment sense.

To give an idea of the potential magnitude of these problems in passive indexing, we ran a back-test to 1975 comparing an all equity portfolio based on MSCI's market capitalization weights on the one hand, with a second portfolio based on fixed weights, which were close to relative GDP weights. The results are shown in the table below:

1975-2010 Real Returns: Market Cap Weighted Equity Portfolio vs Monthly Rebalance Fixed Weights			
	Fixed Weights	Returns	Sharpe Ratio
Market cap weighted		**6.6%**	0.44
Fixed weight return		**7.3%**	0.48
return improvement for fixed (GDP) weights versus market cap weights		**0.7%**	
Weighted Average of constituent returns		**6.8%**	
US	27%	7.0%	
Europe	26%	7.2%	
UK	5%	9.4%	
Japan	13%	4.5%	
Pacific x-Japan	4%	8.2%	
Canada	3%	6.7%	
Emerging Market	23%	6.8%	
	100.0%		

The fixed-weight buy and hold strategy had an annualized return of 7.54% while a market cap weight buy and hold strategy return only 6.32% annually. This is a significant difference considering that they are both passive, buy and hold strategies. The Sharpe Ratio for the fixed weight was also higher. This confirms that the market capitalization weighted approach, practiced by many investors and passive index funds, often underperforms a more simple fixed-weight buy and hold approach with periodic rebalancing. There has been quite a lot written on this issue recently, including an excellent book called The Fundamental Index by Harry Markowitz, Robert Arnott, et al.

The three portfolios we used were described in the body of the paper, section 3, but we will list them here again for ease of reference:

US portfolio (1926-2010):

- 60% US equities
- 20% US 10 year Treasury notes
- 10% US investment grade corporate bonds
- 10% US 3 month T-bills

and

1926-2010: Three country			1975-2010: Broadly diversified	
30%	US Equities			Equities
20%	UK Equities		10.0%	US broad
10%	Japanese Equities		10.0%	US value
20%	US Investment Grade Corporate bonds		4.5%	Europe broad
10%	US 10 year Treasury note		4.5%	Europe value
10%	US 3 month T-bills		2.5%	UK broad
			2.5%	UK value
100%	Total		3.5%	Japan broad
			3.5%	Japan value
			2.0%	Pacific x-Japan broad
			2.0%	Pacific x-Japan value
			1.0%	Canada broad
			1.0%	Canada value
			8.0%	Emerging Market broad
			10.0%	US 10 year Treasury note
			5.0%	US Investment Grade Corporate bonds
			10.0%	Real Estate (REITs)
			10.0%	Commodities (GSCI)
			10.0%	US 3 month T-bills
			100%	Total

In what follows, we will explore, in more detail, the return and risk characteristics of these buy and hold portfolios. In the case of the US portfolio, we calculated the total inflation adjusted return over two time periods: 1) 1926-2010, and 2) 1975-2010.

The table below shows the historical return of this simple portfolio over those two time periods:

1926-2010: US Buy and Hold Portfolio	weights	returns
US equities	60%	6.4%
US Treasury 10 year notes	20%	2.2%
US corporate bonds	10%	3.1%
US 3-month T-bills	10%	0.8%
Weighted average return		4.7%
Actual return of portfolio		**5.1%**
Extra return from rebalaning		0.4%
1975-2010		
US equities	60%	7.0%
US Treasury 10 year notes	20%	4.1%
US corporate bonds	10%	5.2%
US 3-month T-bills	10%	1.6%
Weighted average return		5.7%
Actual return of portfolio		**6.0%**
Extra return from rebalancing		0.3%

Notice that the return on the actual portfolio is higher by about 0.4% than what we would have estimated taking the returns of each asset and multiplying by their weights in the portfolio. The source of this 0.4% extra yield is the monthly rebalancing which takes advantage of both mean reversion and diversification[77].

While 0.4% does not seem like much incremental return, over the long run it makes a very big difference to total accumulated wealth. For instance, a portfolio growing at 5.1% for 84 years (1926-2010) winds up accumulating to 40% more than a portfolio growing at 4.7%, even though the return of 5.1% is less than 10% greater than 4.7%. In long term investing, every 0.25% is worth a lot in terms of long term capital growth.

We used two other more diversified portfolios for the historical studies as described below:

1926-2010: Three country					**1975-2010: Broadly diversified**			
30%		US Equities				Equities		
20%		UK Equities			10.0%	US broad		
10%		Japanese Equities			10.0%	US value		
20%		US Investment Grade Corporate bonds			4.5%	Europe broad		
10%		US 10 year Treasury note			4.5%	Europe value		
10%		US 3 month T-bills			2.5%	UK broad		
					2.5%	UK value		
100%		Total			3.5%	Japan broad		
					3.5%	Japan value		
					2.0%	Pacific x-Japan broad		
					2.0%	Pacific x-Japan value		
					1.0%	Canada broad		
					1.0%	Canada value		
					8.0%	Emerging Market broad		
					10.0%	US 10 year Treasury note		
					5.0%	US Investment Grade Corporate bonds		
					10.0%	Real Estate (REITs)		
					10.0%	Commodities (GSCI)		
					10.0%	US 3 month T-bills		
					100%	Total		

[77] There is a full literature on this interesting and valuable effect. To see the effect that comes from diversification (and why the effect has been termed the "diversification return" by other writers) consider the simple case of a portfolio consisting of two assets which each has a zero return, but which are perfectly negatively correlated. The table below shows how combining these two assets gives a portfolio that returns 2.5% even though each of the constituents has a zero return.

time period	Return on Asset 1	Return on Asset 2	Equally weighted portfolio return
1	25.00%	-20.00%	2.50%
2	-20.00%	25.00%	2.50%
3	25.00%	-20.00%	2.50%
4	-20.00%	25.00%	2.50%
total return	**0.00%**	**0.00%**	**2.50%**

Another interesting example of the benefit of diversification comes from analyzing the three country portfolio, and comparing it both to its constituent returns and also to the return of the less diversified US portfolio, as in the table below:

1926-2010: 3 Country Buy and Hold Portfolio		
US equities	30%	6.4%
UK equities in $s	20%	5.9%
Japanese equities in $s	10%	3.4%
US Treasury 10 year notes	20%	2.2%
US corporate bonds	10%	3.1%
US 3-month T-bills	10%	0.8%
Weighted average return		4.3%
Actual return of portfolio		**5.2%**
Extra return from rebalancing		1.0%
3 country equity portfolio		
US equities	30%	6.4%
US Treasury 10 year notes	20%	5.9%
US corporate bonds	10%	3.4%
weighted average of 3 country portfolio		**5.7%**

Notice here that the difference between the actual return of the portfolio and the simple weighted average of the constituent returns is 1.0%, much greater than in the case of the US only portfolio which was 0.4%. It is also interesting that the return of the 3 Country portfolio at 5.2% was actually a bit higher than the return on the US only portfolio over the same period of 5.1% despite the substantial drag of the poor Japanese equity performance. These are illustrations of some of the more subtle benefits of diversification and simple portfolio rebalancing.[78]

[78] While on the topic of Japanese equities, which lost over 95% of their real value over the period of WWII, let us address an aspect of long term investing that is often found a bit surprising. The thought experiment is this: imagine in 1925 that you invested ½ of your wealth in US equities and the other ½ of your wealth in various equities (Russian, German, Austrian, Japanese) which (and this is not what happened historically) immediately lost 100% of their value. Let's say that US equities then had a return of 6.4% over the following 85 years to the present (which they did).

What would be your portfolio return? Since you lost ½ of your portfolio at the start, would your return be ½ of the US return of 3.2%? In fact, your return would be 5.5%, or "only" 0.9% lower. The calculation is easy enough:

{0.5 * (1 + .064) ^ 85 } ^ (1/85) -1

Where 85 is the number of years, and 0.5 represents the initial loss of 50% of your capital. Perhaps the most interesting realization in this case is to come full circle and to realize that actually, 0.9% is a lot of return, as it makes the difference between $100 growing into roughly $20,000 versus $100 growing into $10,000, exactly ½ of the ending wealth, as ½ of the starting wealth was lost. We hope we are not confusing matters in discussing this here (although it is after all just a footnote), as we have sidestepped the issue of monthly rebalancing in this case, in order to focus on another aspect of long term investing.

2) Portfolio Construction

This study examines the monthly total returns of a number of assets in four separate portfolios over a different time horizon. We began with nominal returns for each asset. Using the CPI Index from the US Bureau of Labor, we then discounted all assets by the rate of inflation to derive "real" returns for each asset. We then discounted each asset by the return on cash, specifically 90-day US Treasury Bills, to arrive at the "excess" real return for each asset, which was easier to work with in constructing portfolios. The excess real return for each asset, for each monthly period, is then multiplied by its respective weight.

Each portfolio is initiated on its inception date with a baseline allocation corresponding to what we believe would be a common balanced portfolio asset allocation of an investor with moderate risk aversion. Those baseline allocations were described in more detail in section 2 above. The resultant weighted returns are then summed together in each time period *t*. Finally, we add the return of cash back in to the portfolio thus giving the real return:

$$R_t \{\text{portfolio}\} = \sum_i W_{i_t} * R_{i_t} + r_t$$

There are many instances throughout the study where the sum of the desired weights is greater than 1. Since we decided at the outset not to employ leverage we scale down all of the assets equally as given by the formula below to create unlevered weights:

$$w_i^u = w_i \ / \ (\sum_1^n w_i)$$

Other instances occur when the sum of the assets is less than 1, and in those cases the balance is assumed to be held in cash and earns the 3month T-bill return, but just for a month. We gave consideration to using a more complex portfolio optimization scheme taking account more explicitly of expected returns and the covariance of the assets, but decided against it in the interest of testing a very simple, easily implementable strategy. Also, we felt that using an optimization approach would inevitably lead to incremental data mining, as it would introduce further degrees of freedom in our historical study. We did not test any portfolio construction technique than the simple method explained above.

3) More detailed description of data used (...still work in progress)

<u>*Equities:*</u> Our equity data comes primarily from MSCI, which was sourced from Datastream. Value metrics such as P/E and P/B come primarily from MSCI as well. MSCI has comprehensive data for the major markets back to 1970, however their value indices start in 1975. The only exception is the Emerging Markets Index, which did not start until 1988. Since a suitable EM equity index did not exist before 1988 we created our own using indices and data from Global Financial Data (see appendix).

For our longer studies we used a variety of sources for equity data. For the US equity market we used the S&P 500 beginning in 1926. In 1975 we switch to the MSCI Index, which is more comprehensive and matches the other international indices. For the UK we use the FTSE 100 from 1926, again switching to MSCI in 1975. For Japan we use the TOPIX form 1926 and again switch over to MSCI in 1975. For the other four regions (Europe ex UK, Canada, Japan ex Pacific and Emerging Markets) we use only MSCI data beginning in 1975. We checked our long term return indices against results from Triumph of the Optimists (Dimson, Marsh, Staunton) and our data seemed to match up reasonably well.

US Sovereign Debt:

The 10 year US rate in all our return series comes from Robert Shiller's website. Here is the definition he gives in Irrational Exuberance:

From 1890 to 1953 the interest rate series is from Sidney Homer, *A History of Interest Rates*. (New Brunswick, N.J.: Rutgers University Press, 1963), tables 38 (col. 3), 45 (col. 15), 48(col. 1), and 50(col.1), interpolated from annual to monthly. For April 1953 through 2004 it is the monthly ten-year Treasury Bond Yield, from the Federal Reserve. This long rate series is very similar to the one in Jeremy J. Siegel, *Stocks for the Long Run*, 3rd ed. (New York: McGraw-Hill, 2002), also available at http://jeremysiegel.com. Siegel describes the series in Jeremy J. Siegel, "The Real Rate of Interest from 1800-1990: A Study of the U.S. and the U.K.," *Journal of Monetary Economics*, 29(1992): 227-52.

Link to appendix of Irrational Exuberance in Google books:
http://books.google.com/books?id=x1PaZY_KtBEC&pg=PA235&lpg=PA235&dq=robert+shiller+sidney+homer+description+interest+rates&source=bl&ots=CQrtgYHzxJ&sig=KHtnJ4Xv1qrzudma6PM4ef277z8&hl=en&ei=NCPaTKy2J4O8lQfEy6zVCQ&sa=X&oi=book_result&ct=result&resnum=1&ved=0CBMQ6AEwAA#v=onepage&q&f=false

The monthly ten-year yield from the Federal Reserve that Shiller references can be found here:

http://www.federalreserve.gov/releases/h15/data/Monthly/H15_TCMNOM_Y10.txt

For the greater than 10 year rate, which gets used to determine the spread of BAA corporate bonds over US Treasuries, we use the following:

1/31/1920 – 12/31/1924 - Nominal 10-year yield – Shiller {per Homer}
1/31/1925 – 4/29/1953 – Composite 10-year yield - FED
5/31/1953 – 1/31/1977- Constant Maturity 10-year yield – GFD, {FED Also}
2/28/1977 – Present – Constant Maturity 30-year yield {GFD, Bloomberg {FED Also}
(The latter two from above are rolled into one series from GFD; see 3. below)

As per above, starting at 1/31/1925 we use the long-term 10-year composite rate provided by the US Federal Reserve. This calculates the unweighted average of rates on all outstanding bonds neither due nor callable in less than 10 years. The last date we use this series is 4/29/1953. It is

not clear whether these are end of month or midmonth data points, although we suspect they are midmonth.

On 5/27/1953 it switches to the 30-year yield as provided by Global Financial Data.

We did not smooth the splicing in the data; when changing data sources we simply used one yield for a given month from source A and the following month used the yield from source B.

http://www.federalreserve.gov/releases/h15/data/Monthly/H15_TCMNOM_Y30.txt

Definition from GFD: Sources: Sydney Homer, A History of Interest Rates, Princeton: Rutgers, 1963 from Joseph G. Martin, Martin's Boston Stock Market, Boston: 1886 (1800-1862), Hunt's Merchants Magazine (1843-1853), The Economist (1854-1861), The Financial Review (1862-1918), Federal Reserve Bank, National Monetary Statistics, New York: FRB, 1941, 1970 (annually thereafter); and Salomon Brothers, Analytical Record of Yields and Yield Spreads, New York: Salomon Brothers, 1995

The Long-term Treasury Bond Index is monthly from February 1862 until December 1933, and weekly thereafter. It is a combination of several indices. From February 1861 until December 1877, the 6% U.S. Government bonds of 1881 are used. From January 1878 until January 1895, the 4% U.S. Government Bonds of 1907 are used, and from February 1895 until December 1918, the 4% U.S. Government Bonds of 1925 were used. Where no trades were recorded during a given month, the previous month's yield was used. The source for this data is William B. Dana Co., The Financial Review, New York: William B. Dana Co. (1872-1921) which reprinted data published by The Commercial and Financial Chronicle. Beginning in 1919, the Federal Reserve Board's 10-15 year Treasury Bond index is used. This is used through 1975. In 1976, the 20-year Bond is used, and beginning on February 26, 1977, the 30-year bond is used. Beginning on February 19, 2002, the 30-year Bond series includes all bonds of 25 or more years.

Data for 1872 through 1918 are taken from the Financial Review. The interest rate series dating back to 1919 are taken from the Federal Reserve, National Monetary Statistics, New York: Federal Reserve Board which was published in 1941, 1970 and annually since then. The Commercial Paper data for 1835 through 1871 are taken from Walter B. Smith and Arthur H. Cole, Fluctuations in American Business, Cambridge: Harvard Univ. Press, 1935, and the Broker Call money data are taken from F. R. Macaulay, The Movements of Interest Rates, Bond Yield, and Stock Prices in the United States since 1856, New York: National Bureau of Economic Research, 1938.

Credit: We use the average of two of Moody's BAA Yield indices from GFD. A 50/50 blend between the BAA Corporate and the BAA Industrial yield is used. We subtract the long US Treasury rate, as described above to arrive at the credit spread for each point in time. We then take that credit spread, and add it to the 10 year Treasury rate series to then arrive at a proxy for a 10 year BAA corporate rate. To create the total return series, we then take that rate, assuming a maturity of 10 years and assuming a constant per period credit loss of 50% of the credit spread, and generate the period by period return, recognizing that credit losses do not arrive so regularly, constantly and even at that assumed rate on average. The long term inflation adjusted returns to BAA credit came

out roughly in line with what is shown in other sources, so we are comfortable with the 50% loss assumption.

Real Estate: Our Real Estate data comes from the FTS NAREIT All REIT's Index, which is a free float adjusted market capitalization weighted index that includes all tax qualified REIT's listed on the NYSE, AMEX and NASDAQ National Market. This data was sourced from Bloomberg.

Commodities: For Commodities Data we use the Goldman Sachs Commodity Index, which starts in 1970.We use both the spot and total return indices and source the data from Bloomberg.

Value equities: MSCI's methodology for segmenting and constructing growth and value indices for each equity market is fairly complex and can be found in full at:
http://www.mscibarra.com/products/indices/international_equity_indices/gimi/vg/methodology.html

Basically, our understanding is that MSCI uses 3 Factors for value versus growth equities segmentation:

a. Book value to price ratio (BV / P)
b. 12-month forward earnings to price ratio (E fwd / P)
c. Dividend yield (D / P)

Cash: The 90-Day US Treasury Bill rate is used for the return on cash. This extends back to the earliest points of our study and was sourced from Global Financial Data

Inflation: For inflation adjusting cash flows we used the US CPI series, which we took from Robert Shiller's website and also checked against the BLS website.

For 10 year inflation expectations, which we use to estimate the expected real return on 10 year US Treasury notes, we use a series provided by Antti Ilmanen. This series is spliced together from various sources. Actual survey data on long-term inflation start in 1978, with Hoey (private collection passed to Fed), Blue Chip Economic Indicators, Survey of Professional Investors and Consensus Economics conducting surveys at different times. From this point, the series is an average of the 2-3 available surveys. (not included is the University of Michigan's inflation surveys that are based on consumer opinions rather than those of professional economists.) Between 1955 and 1978 a statistical estimate of long-term inflation expectations by Kozicki-Tinsley (2006) is used. Their "Survey-Based Estimates of the Term Structure of Expected U.S. Inflation" goes beyond an exponentially weighted average of past inflation rates because it also uses information in consensus forecasts of next-year inflation. After 1978 the estimates' contours nicely match those of available long-term consensus forecasts. Before 1955 Antti uses his own time series estimates, combining two methodologies. One is just taking a weighted average of past 10-year inflation rates, with gradually declining weights. Until U.S. abandoned the Gold Standard in 1933 this time series estimate with zero was averaged with zero (thereby anchoring the inflation forecast) based on the idea that amidst Gold Standards inflations and deflations were equally likely to occur and not likely to become sustained. The other of the two methodologies is a regression of the next-decade inflation on past-decade inflation and using the fitted value as expected inflation estimate. Inflation data since 1870 is used and the coefficients are re-estimated

each year with new data. This approach has very few independent observations, so it is only started in 1930s. Both approaches give broadly similar contours and later evidence suggests that survey expectations tend to be adaptive (reflecting past inflation experience, with declining weights). However, it seems reasonable that the knowledge of Gold Standard and the experience of near-zero inflation over very long histories anchored expectations better to zero until the Gold Standard was abandoned.

For many assets, we were forced to splice data over periods of multiple months (but never years) during periods where it was unavailable, most commonly due to World War II.

Note: further notes about the data and calculations are contained within the spreadsheet on first worksheet page titled "notes"

Some extra charts to potentially include?